Signal and System

Chapter 1

Signals and Systems

Introduction:

❑ SIGNALS:

A signal is a set of data or information. Examples include a telephone or a television signal. We will deal almost exclusively with signals that are functions of time.

❑ SYSTEMS

A system is an entity that processes a set of signals **(inputs)** to yield another set of signals **(outputs)**. A system may be made up of physical components, as in electrical, mechanical, or hydraulic systems **(hardware realization)**, or it may be an algorithm that computes an output from an input signal **(software realization)**.

Classifications of Signals

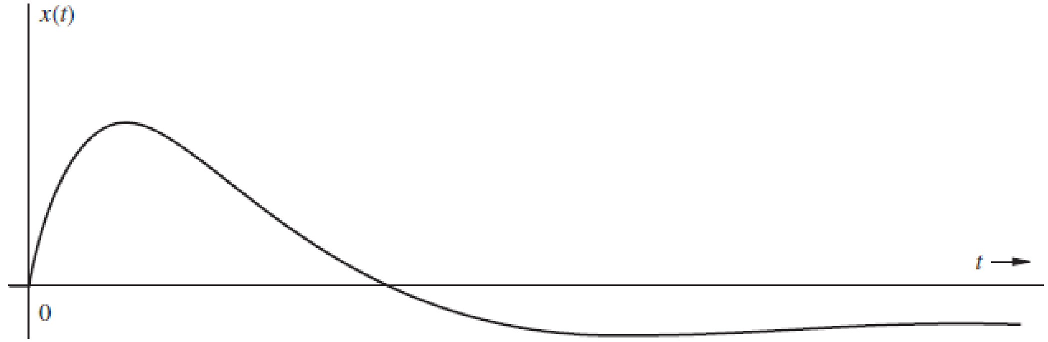
Signals could be classified into these categories:

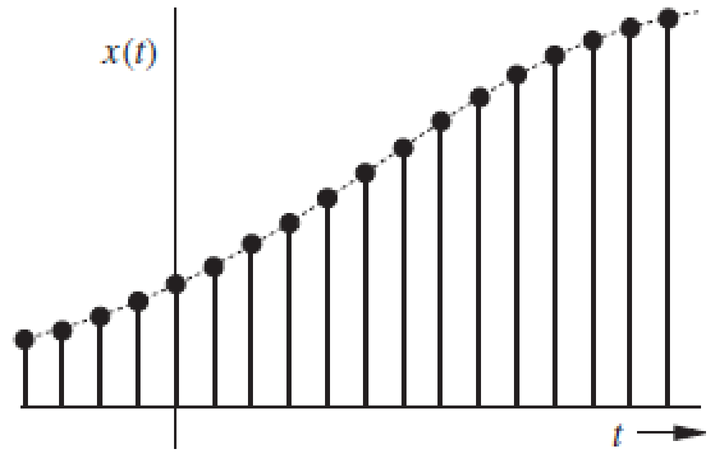
1. Continuous-time and discrete-time signals
2. Analog and digital signals
3. Periodic and aperiodic signals
4. Energy and power signals

Classifications of Signals

1. Continuous-time and discrete-time signals

A signal that is specified for a continuum of values of time t is a continuous-time signal.



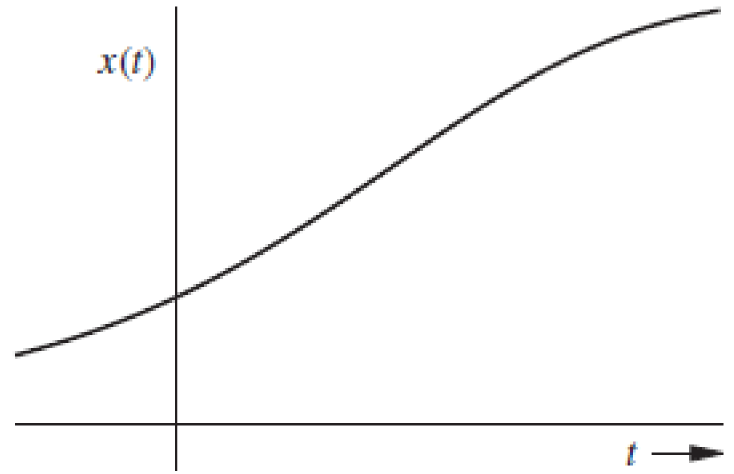


A signal that is specified only at discrete values of t is a discrete-time signal.

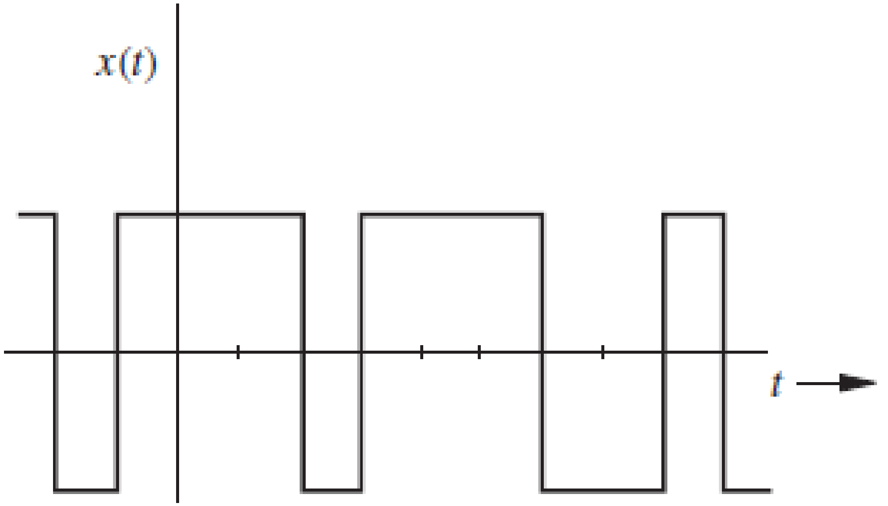
Classifications of Signals

2. Analog and digital signals

A signal whose amplitude can take on any value in a continuous range is an analog signal. This means that an analog signal amplitude can take on an infinite number of values.



A digital signal, on the other hand, is one whose amplitude can take on only a finite number of values.



Classifications of Signals

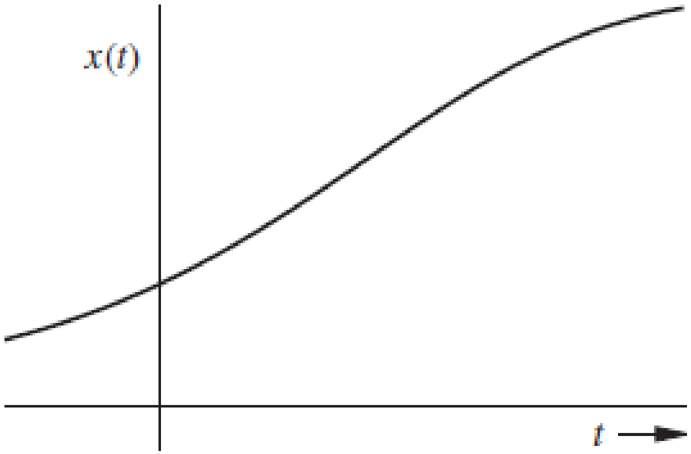
2. Analog and digital signals

- Signals associated with a digital computer are digital because they take on only two values (binary signals). A digital signal whose amplitudes can take on M values is an M-ary signal.

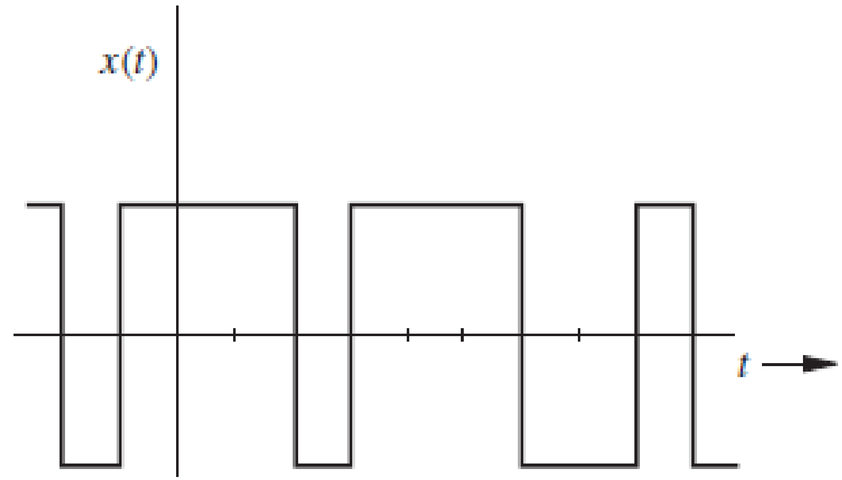
- The terms continuous time and discrete time qualify the nature of a signal along the time (horizontal) axis. The terms analog and digital, on the other hand, qualify the nature of the signal amplitude (vertical axis).

Classifications of Signals

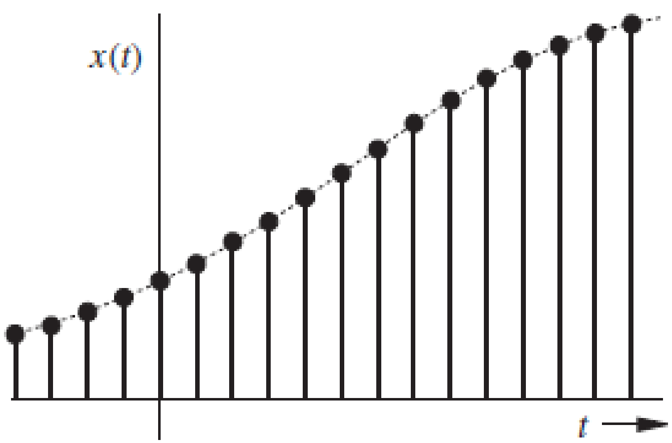
2. Analog and digital signals



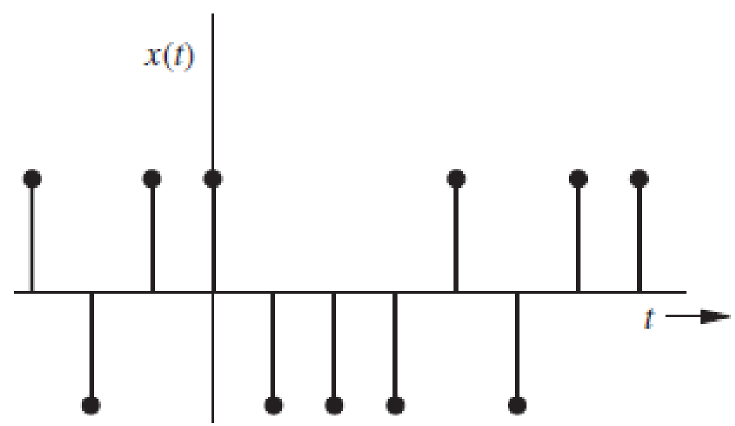
Analog Continuous signal



Digital Continuous signal



Analog Discrete signal



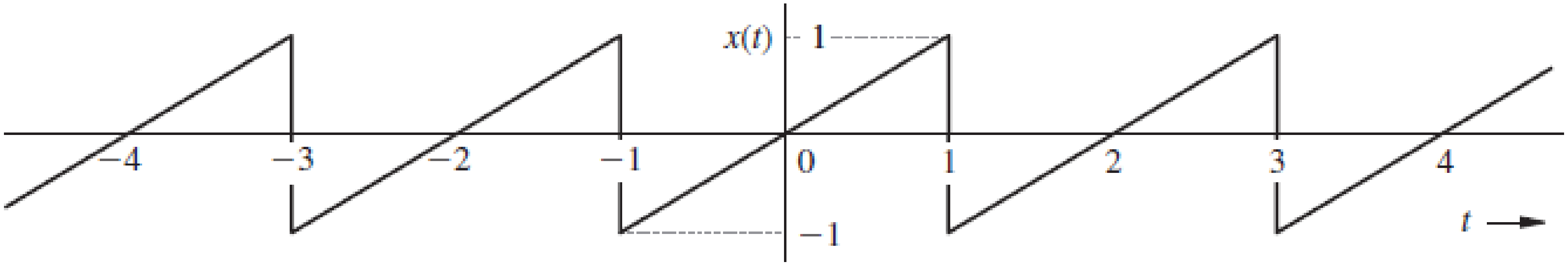
Digital Discrete signal

Classifications of Signals

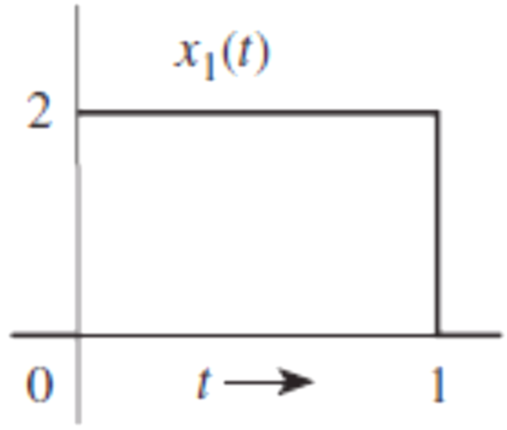
3. Periodic and aperiodic signals

A signal x(t) is said to be periodic if for some positive constant T_0:

$$\boldsymbol{x(t) = x(t + T_0) \qquad for\ all\ t}$$



Period = 2



A signal is aperiodic if it is not periodic.

Classifications of Signals

4. Energy and Power Signals:

- A signal with finite energy is an energy signal, and a signal with finite and nonzero power is a power signal.

- A signal with finite energy has zero power, and a signal with finite power has infinite energy. Therefore, a signal cannot be both an energy signal and a power signal.

- On the other hand, there are signals that are neither energy nor power signals.

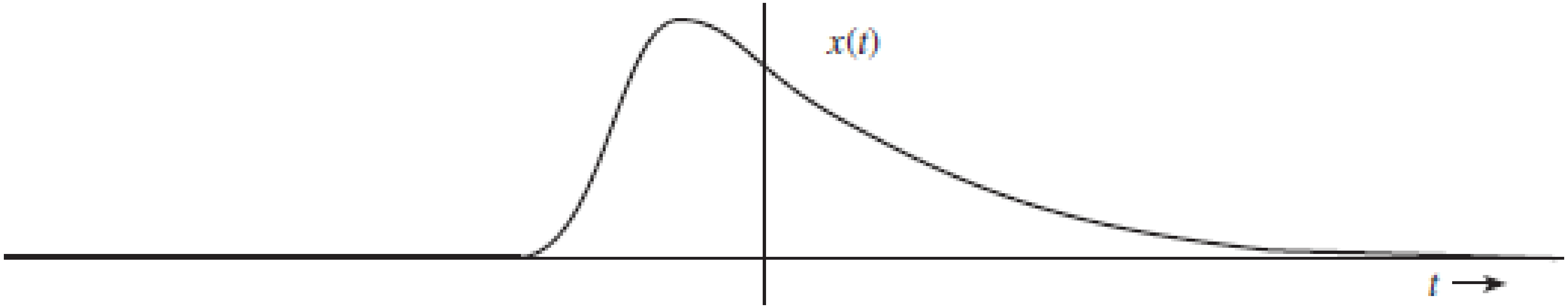
Energy signals and Power Signals

❑ Signal Energy (E_x):

Signal energy (E_x) of a signal x(t) is defined as the area under $|x(t)|^2$,

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Signal energy must be finite for it to be a meaningful measure of signal size. A **necessary condition** for the energy to be finite is that the signal amplitude x(t) → 0 as |t|→∞.

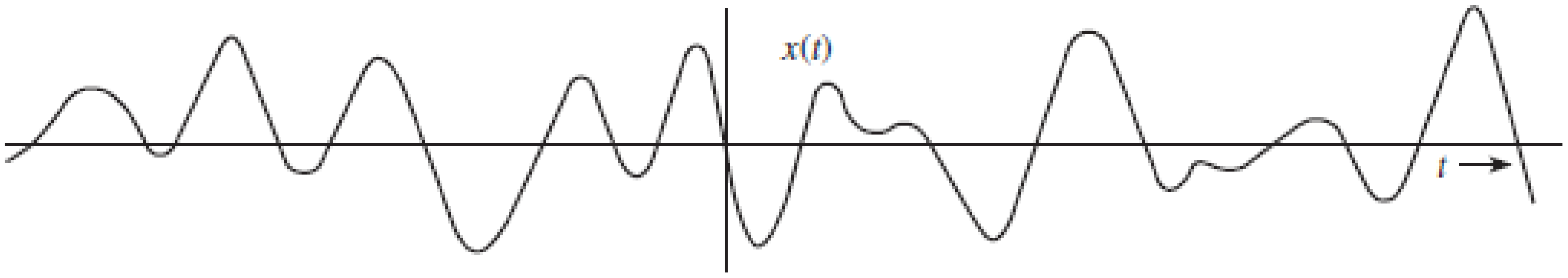


Energy signals and Power Signals

❑ Signal Power (P_x):

When the amplitude of x(t) does not→0 as |t|→∞, the signal energy is infinite. A more meaningful measure of the signal size in such a case would be the power of the signal. For a signal x(t), we define its power P_x as

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

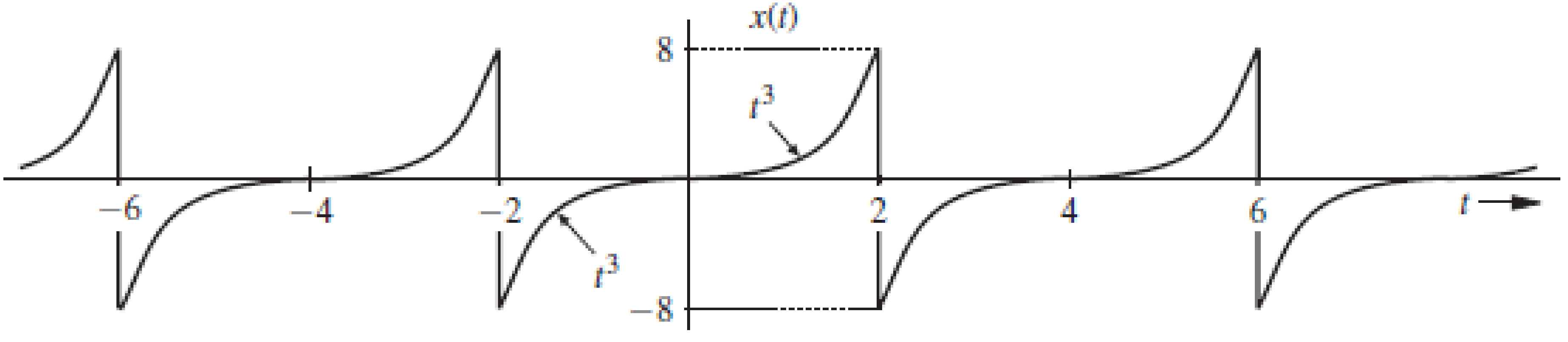


Energy signals and Power Signals

❑ Signal Power (P_x):

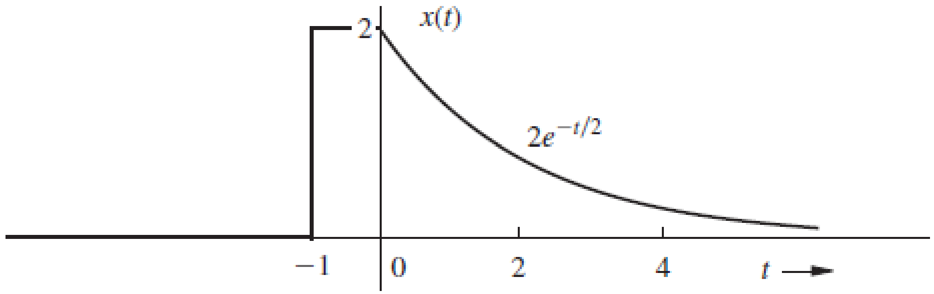
For periodic signals, the power is calculated as:

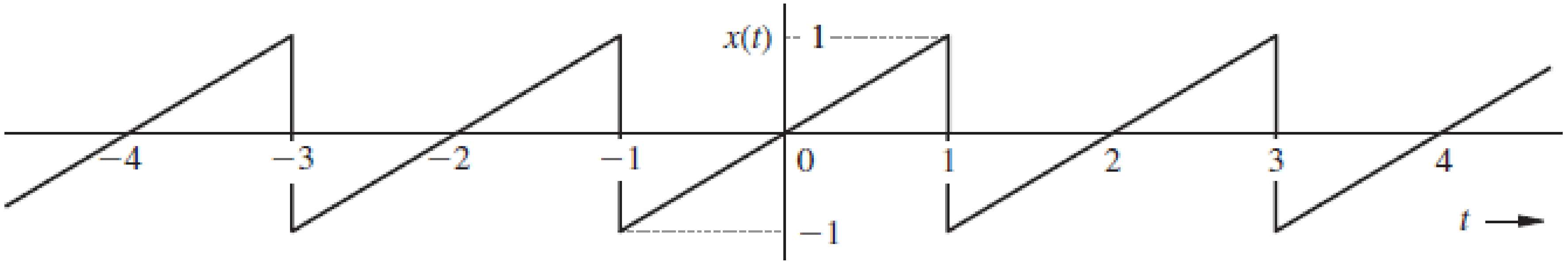
$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$



Example:

Determine the suitable measures (Energy or Power) of the signals shown in figure.

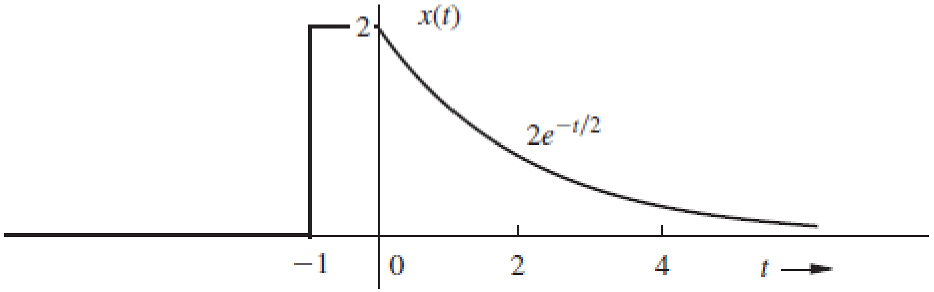




Solution:

This is an energy signal:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$



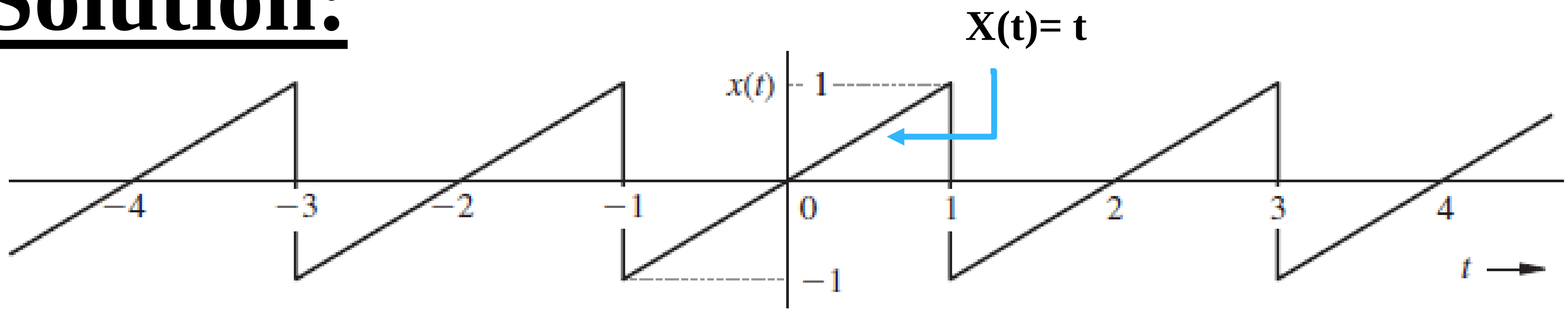
$$E_x = \int_{-1}^{0} 2^2 \, dt + \int_{0}^{\infty} 4e^{-t} \, dt$$

$$E_x = 4 + \left[\frac{4e^{-t}}{-1}\right]_0^{\infty}$$

$$E_x = 4 + \left[\frac{4e^{-\infty} - 4e^{0}}{-1}\right]$$

$$E_x = 4 + 4 = 8$$

Solution:



This is a periodic power signal of period T = 2:

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$P_x = \frac{1}{2}\int_{-1}^{1} t^2 dt$$

$$P_x = \frac{1}{2}\left[\frac{t^3}{3}\right]_{-1}^{1}$$

$$P_x = \frac{1}{2}\left(\frac{1^3}{3} - \frac{(-1)^3}{3}\right)$$

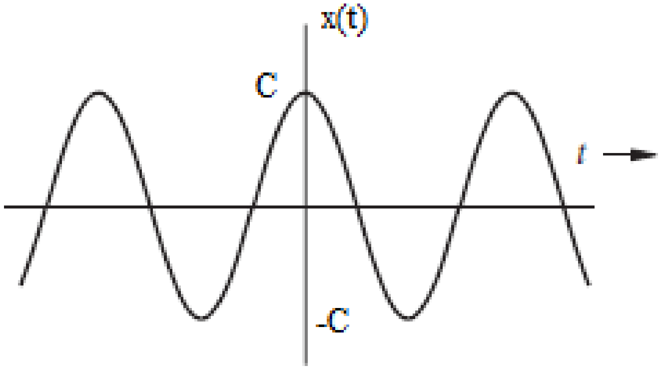
$$P_x = \frac{1}{2}\left(\frac{1}{3} - \frac{-1}{3}\right)$$

$$P_x = \frac{1}{2}\left(\frac{2}{3}\right) = \frac{1}{3}$$

Example:

Determine the power of the signal x(t):

$$x(t) = C\cos(\omega_0 t)$$



Solution:

This signal is a periodic signal of period $T = \dfrac{2\pi}{\omega_0}$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} C^2 Cos^2(\omega_0 t)dt$$

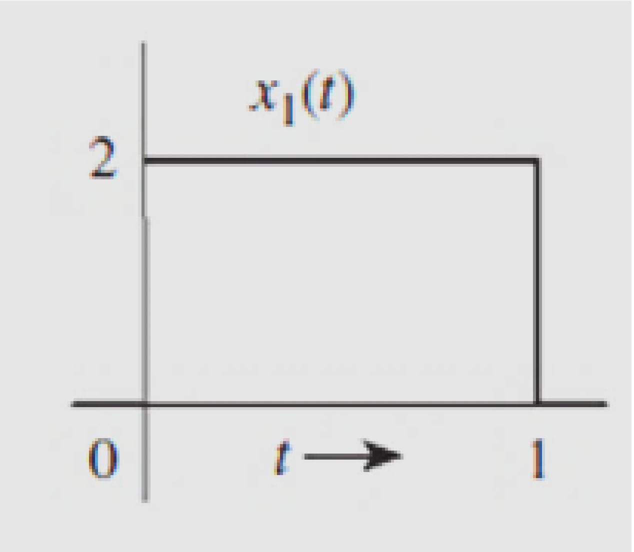
$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} C^2 \left(\frac{1}{2}(1+\cos 2\,\omega_0 t)\right)dt$$

$$P_x = \frac{C^2}{2T}\int_{-T/2}^{T/2} dt$$

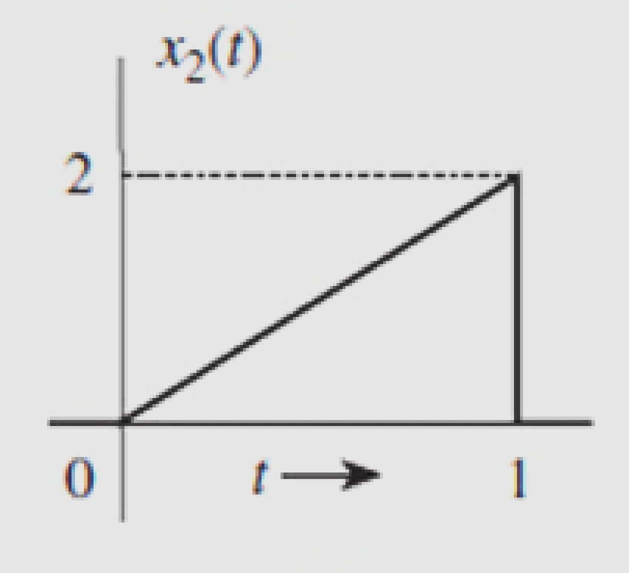
$$P_x = \frac{C^2}{2}$$

Example:

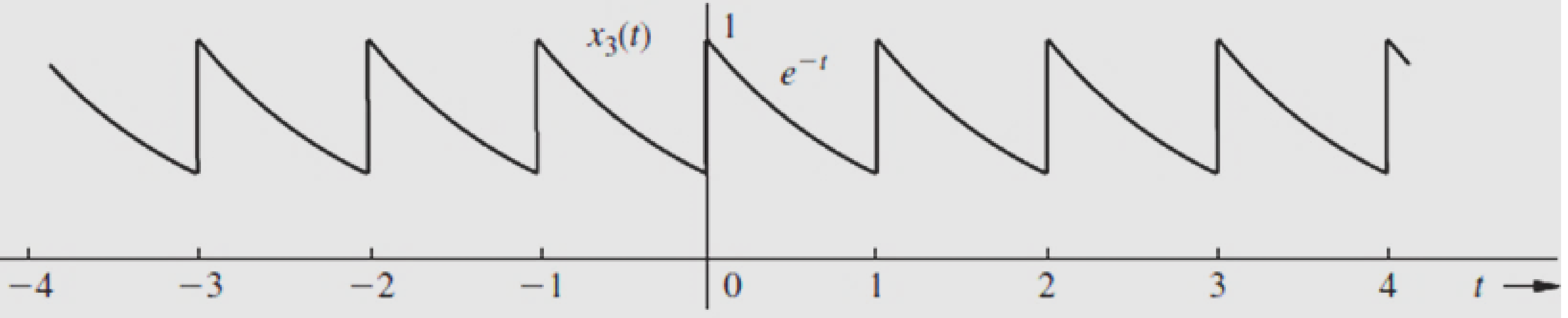
Determine the suitable measure (energy or power) of the shown signals.



(a)

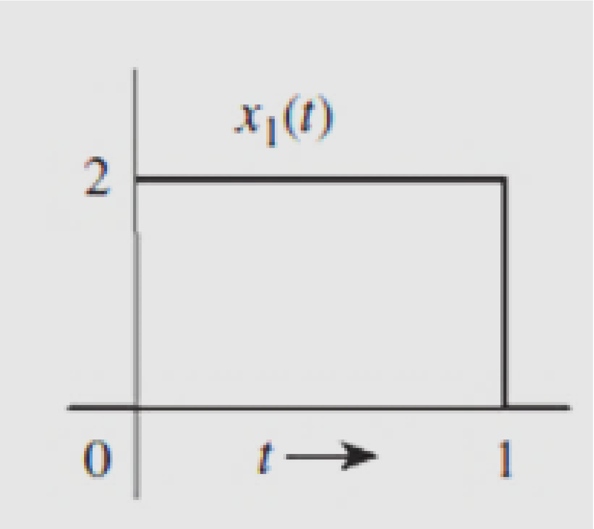


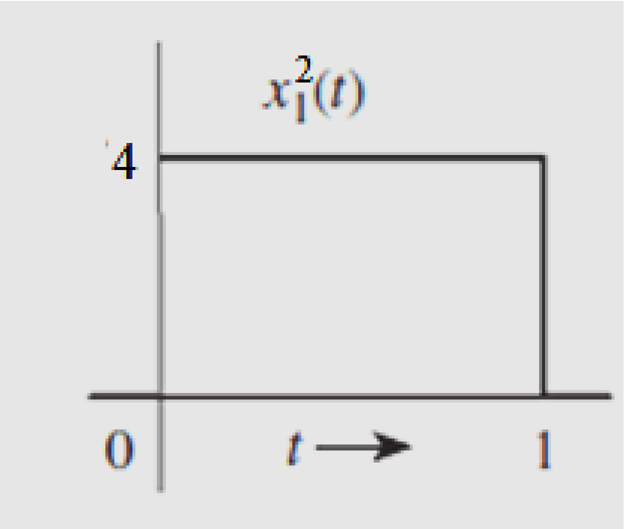
(b)



(c)

Solution:



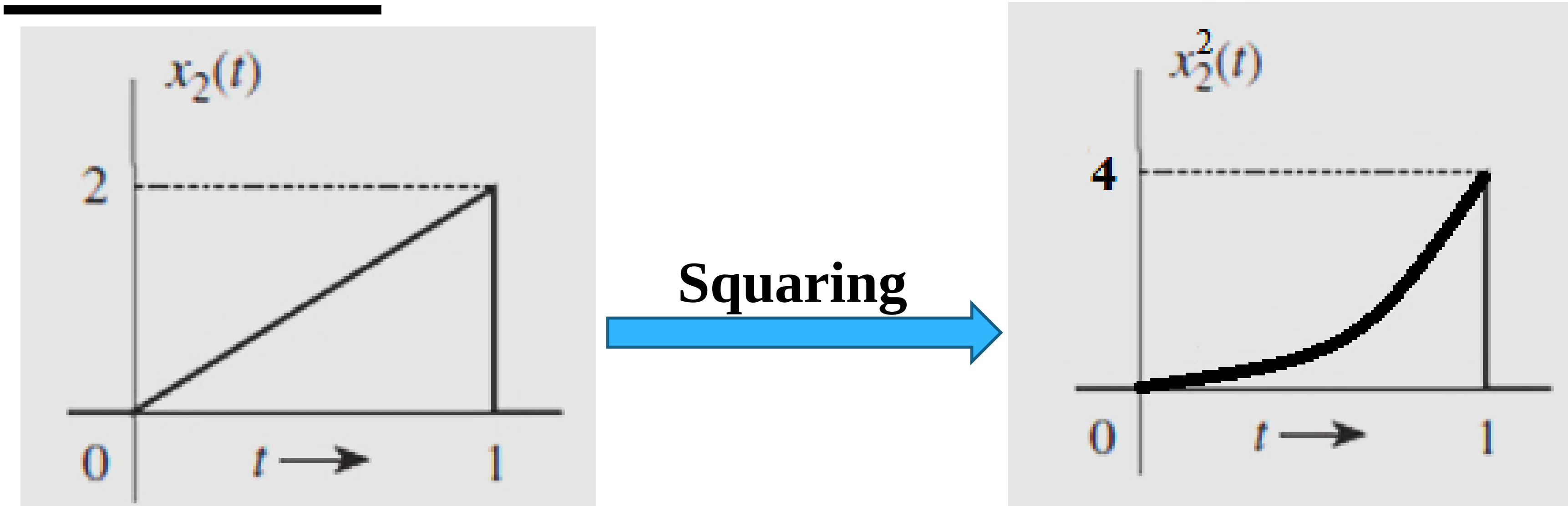


$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E_x = Area\ under\ x^2 curve$$

$$E_x = 4 \times 1 = 4$$

Solution:

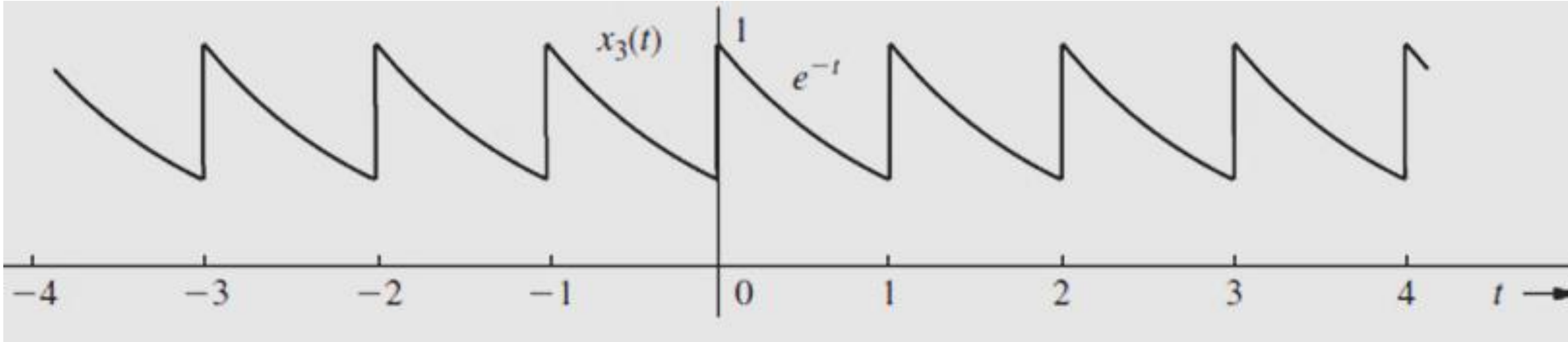


$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E_x = Area\ under\ x^2 curve$$

$$E_x = \frac{1}{3} \times 1 \times 4 = \frac{4}{3}$$

Solution:



$$P_x = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

$$P_x = \frac{1}{1}\int_0^1 e^{-2t} dt$$

$$P_x = \left[\frac{e^{-2t}}{-2}\right]_0^1$$

$$P_x = \left(\frac{e^{-2} - e^{0}}{-2}\right)$$

$$P_x = 0.4323$$

SOME USEFUL SIGNAL OPERATIONS

- We discuss here three useful signal operations: shifting, scaling, and inversion. Since the independent variable in our signal description is time, these operations are discussed as:

1. Time Shifting,
2. Time Scaling, and
3. Time reversal (inversion).

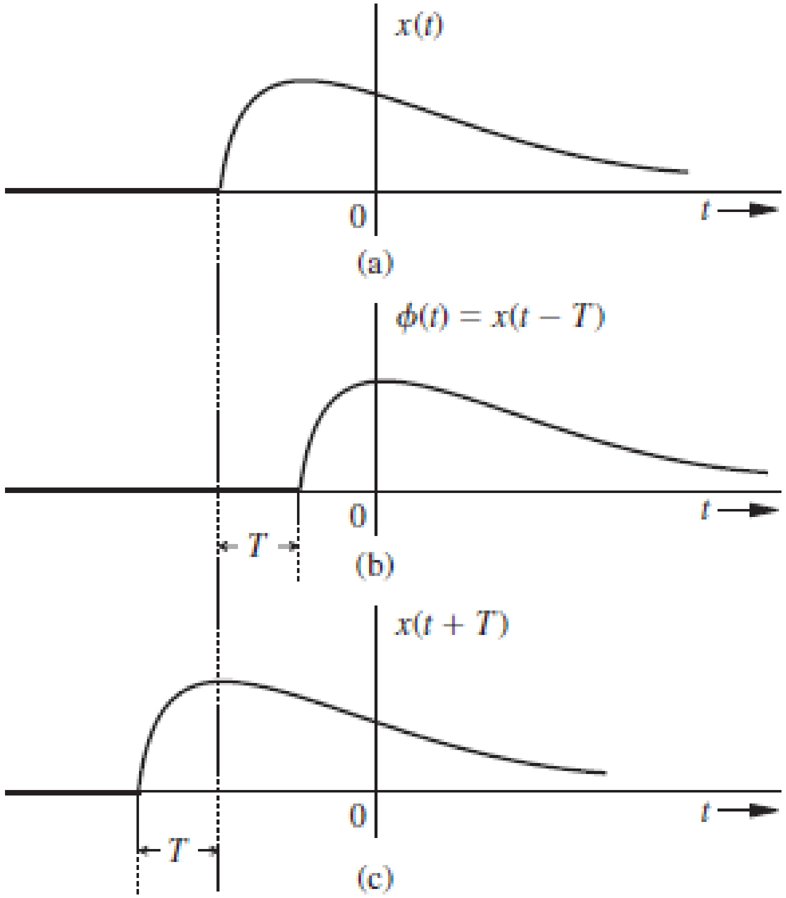
However, this discussion is valid for functions having independent variables other than time (e.g., frequency or distance).

SOME USEFUL SIGNAL OPERATIONS

1. Time Shifting:

Consider a signal x(t) and the same signal delayed by T seconds, which we shall denote by $\phi(t)$.

$$\phi(t) = x(t - T)$$



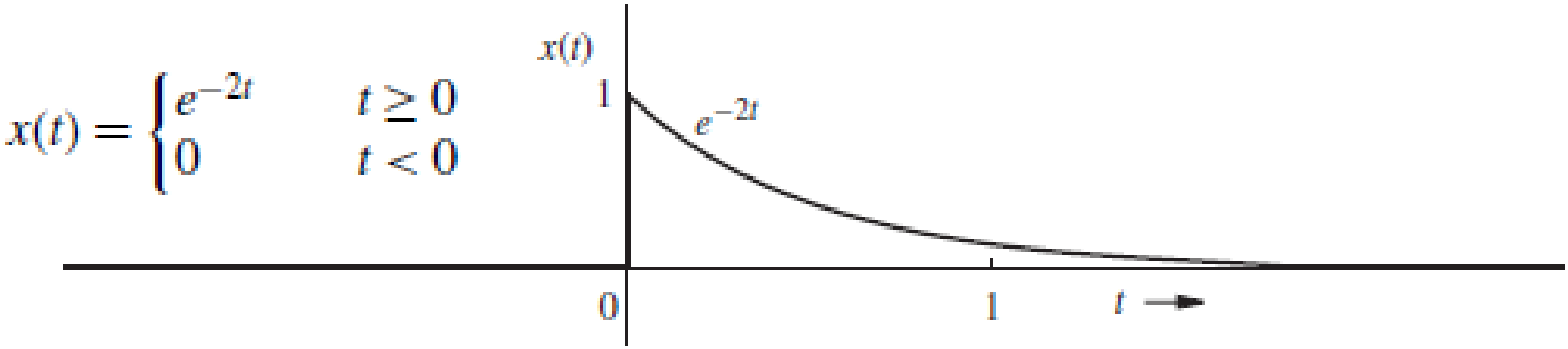
If T is positive, the shift is to the right (delay).
If T is negative, the shift is to the left (advance).

Example:

An exponential function $x(t) = e^{-2t}$ shown in figure is delayed by 1 second.

A. Sketch and mathematically describe the delayed function.

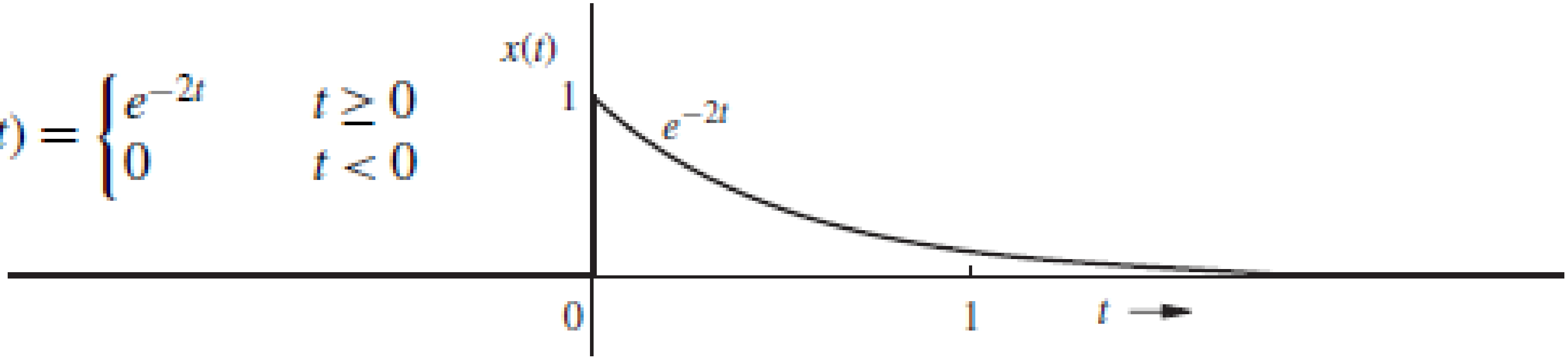
B. Repeat the problem with x(t) advanced by 1 second.



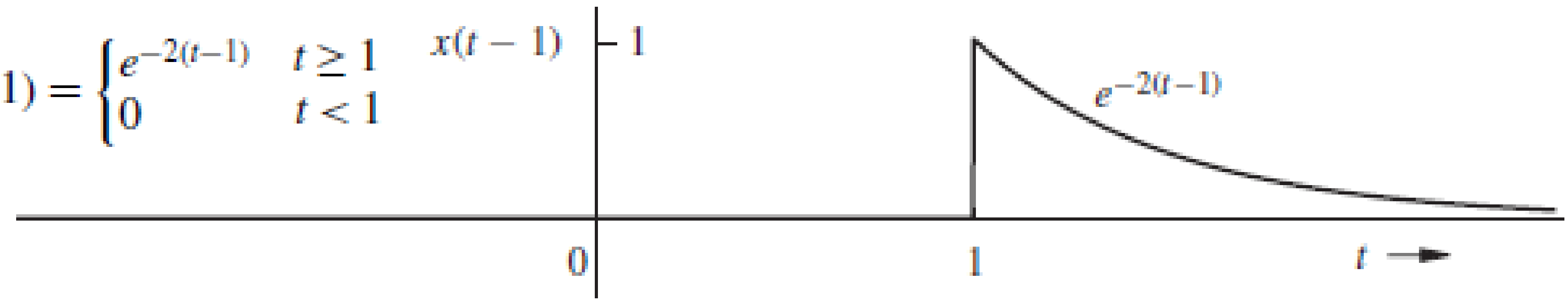
Solution:

A. Delayed function x(t-1).

$$x(t) = \begin{cases} e^{-2t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$



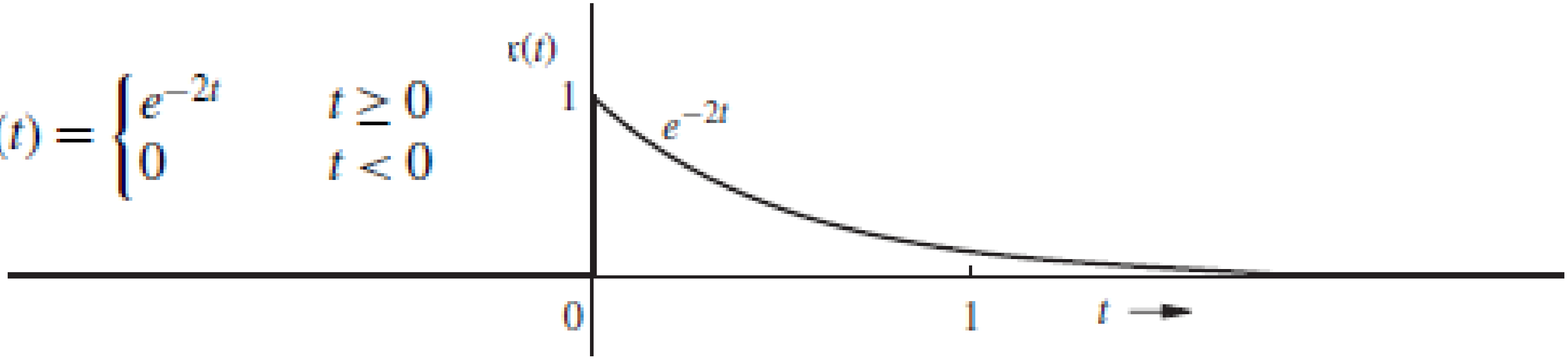
$$x(t-1) = \begin{cases} e^{-2(t-1)} & t \geq 1 \\ 0 & t < 1 \end{cases}$$

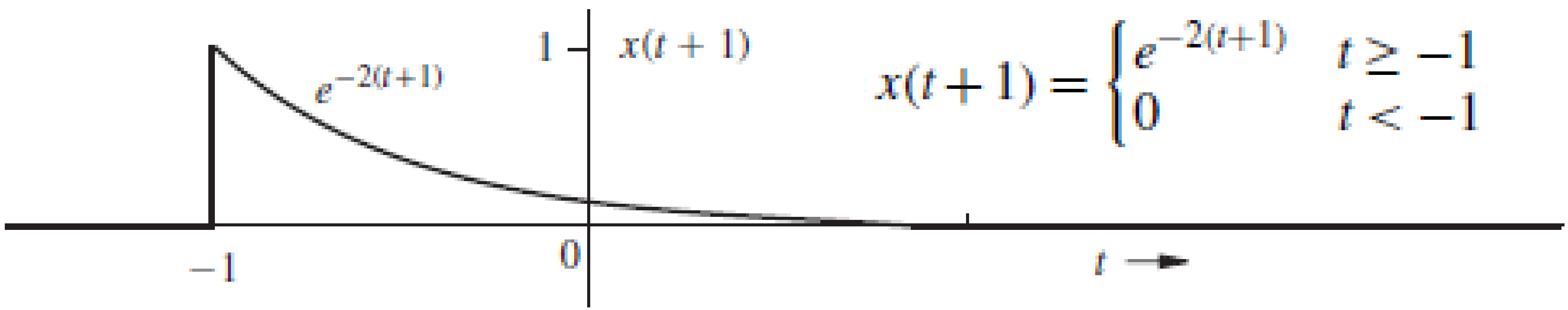


Solution:

B. Advanced function x(t+1).

$$x(t) = \begin{cases} e^{-2t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$



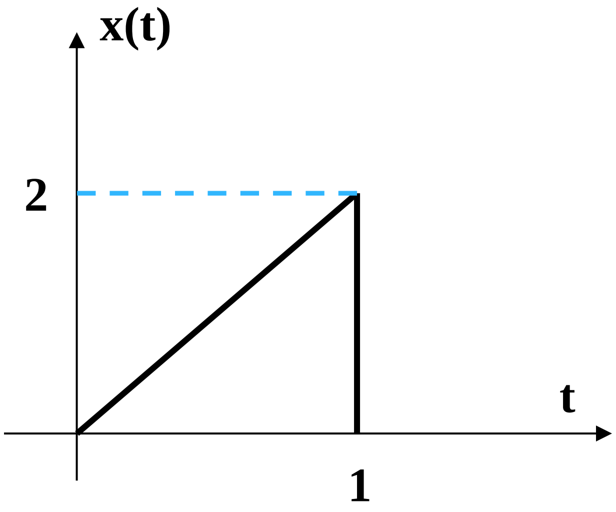


$$x(t+1) = \begin{cases} e^{-2(t+1)} & t \geq -1 \\ 0 & t < -1 \end{cases}$$

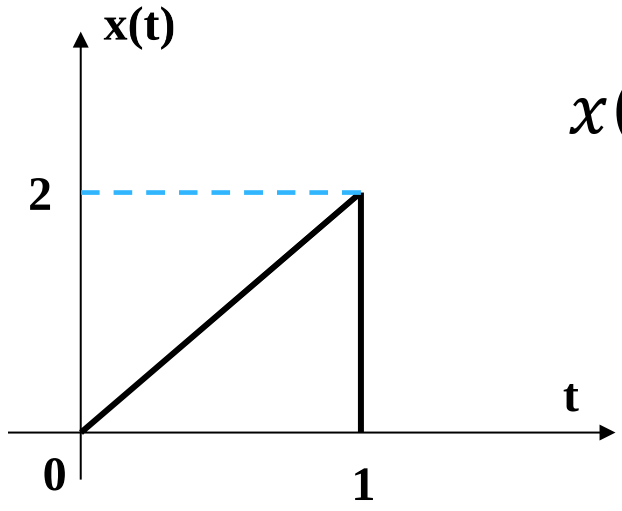
Example:

Write a mathematical description of the signal x(t) shown in the figure.

A. Delay the signal by 2 seconds, then sketch and mathematically describe the delayed function.
B. Repeat the problem with x(t) advanced by 2 second.



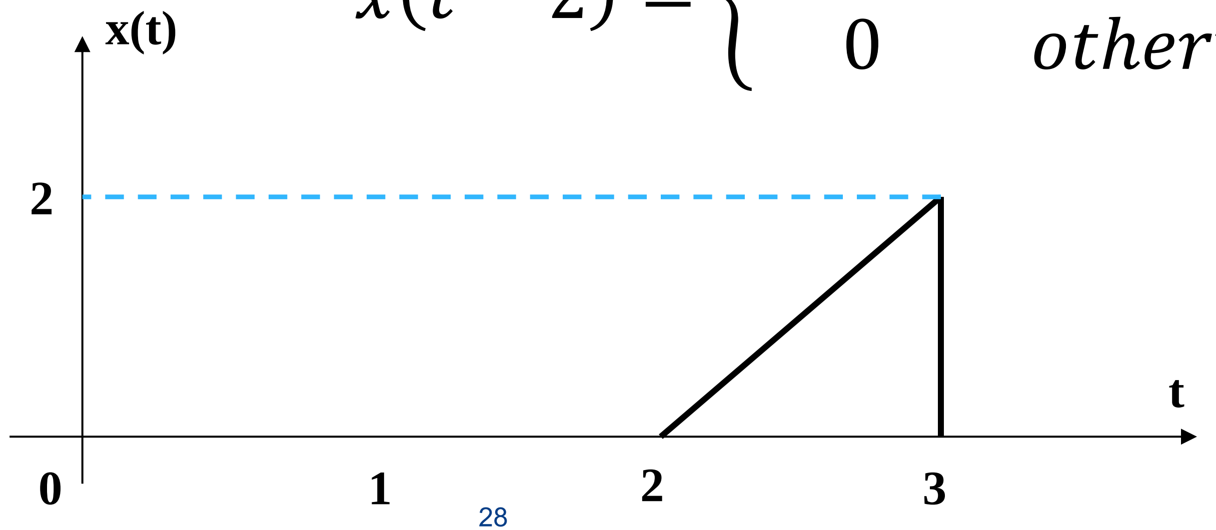
Solution:



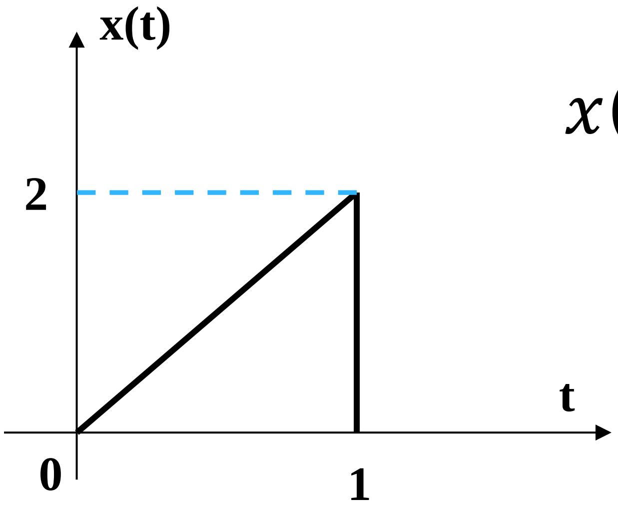
$$x(t) = \begin{cases} 2t & 0 \leq t \leq 1 \\ 0 & otherwise \end{cases}$$

A. Delayed function x(t-2).

$$x(t-2) = \begin{cases} 2(t-2) & 2 \leq t \leq 3 \\ 0 & otherwise \end{cases}$$

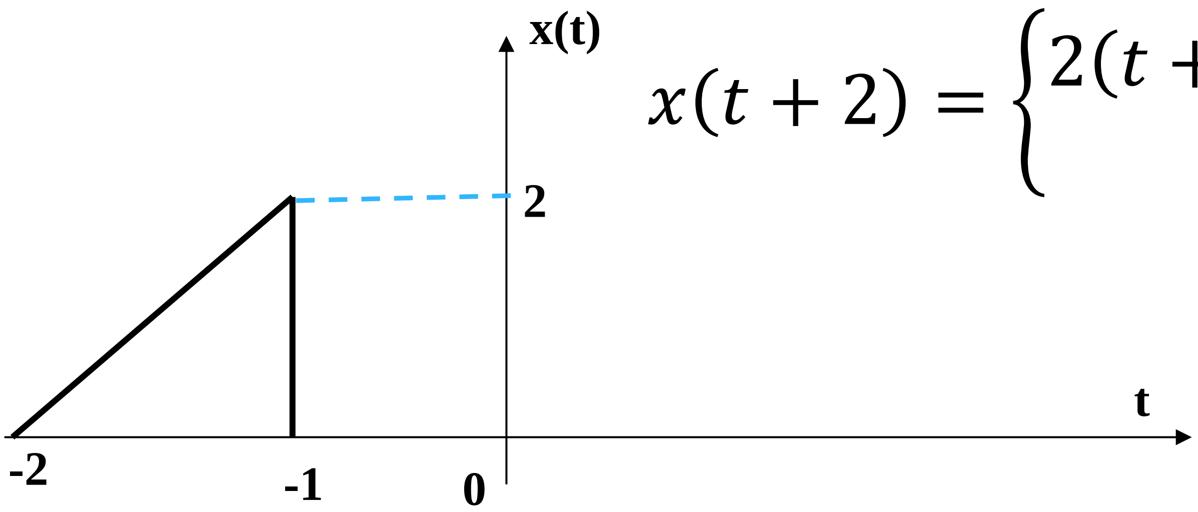


Solution:



$$x(t) = \begin{cases} 2t & 0 \leq \mathrm{t} \leq 1 \\ 0 & otherwise \end{cases}$$

B. Advanced function x(t+2).



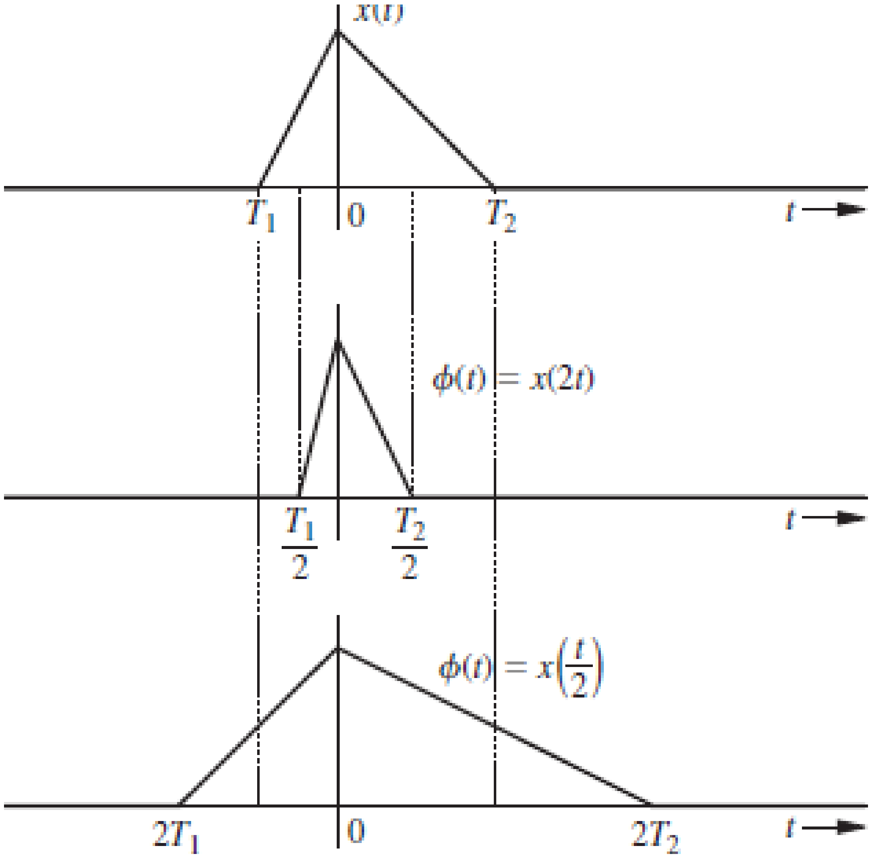
$$x(t+2) = \begin{cases} 2(t+2) & -2 \leq \mathrm{t} \leq -1 \\ 0 & otherwise \end{cases}$$

SOME USEFUL SIGNAL OPERATIONS

2. Time Scaling:

The compression or expansion of a signal in time is known as time scaling. Consider the signal x(t) shown in figure, the time-scaled signal $\phi(t)$ is expressed as:

$$\boldsymbol{\phi(t) = x(at)}$$



x(t) is scaled by a factor of (a).

If a > 1, the signal is compressed

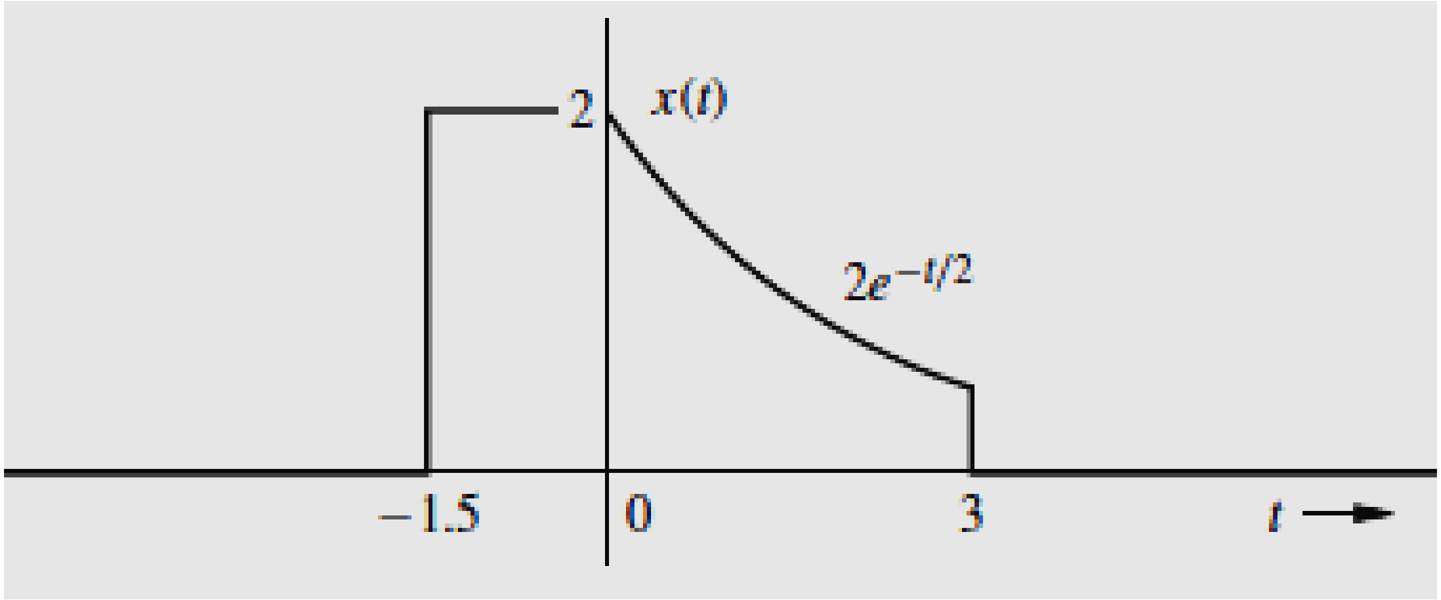
If a < 1, the signal is expanded

Example:

The signal x(t) is shown in figure. Sketch and describe mathematically this signal:

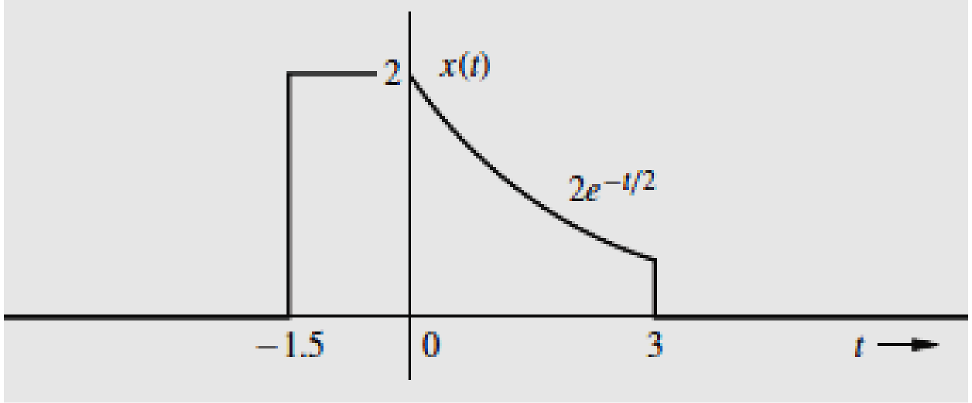
A. Time-compressed by factor 3.

B. **Time-expanded by factor 2.**



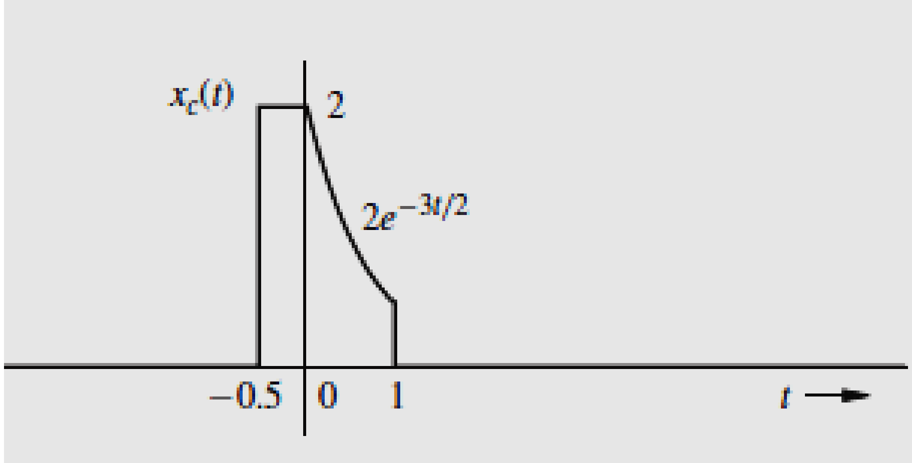
Solution:

$$x(t) = \begin{cases} 2 & -1.5 \leq \mathrm{t} \leq 0 \\ 2e^{-t/2} & 0 \leq \mathrm{t} \leq 3 \\ 0 & otherwise \end{cases}$$



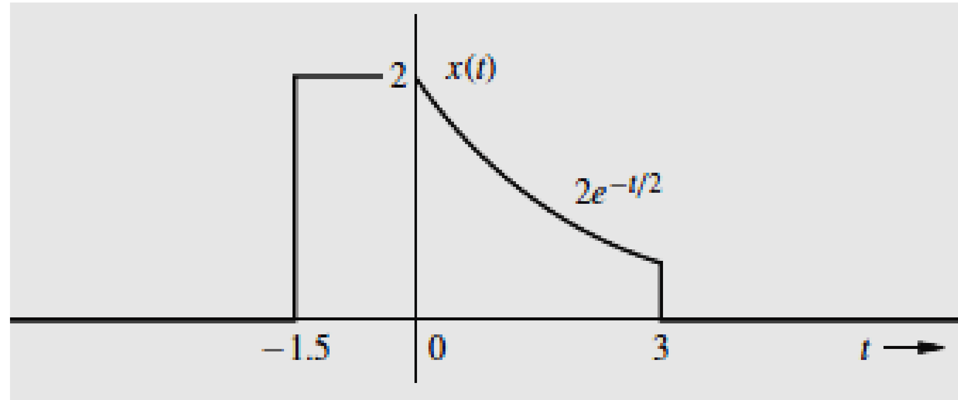
A. Time-compressed by factor 3.

$$x(3t) = \begin{cases} 2 & -0.5 \leq \mathrm{t} \leq 0 \\ 2e^{-3t/2} & 0 \leq \mathrm{t} \leq 1 \\ 0 & otherwise \end{cases}$$



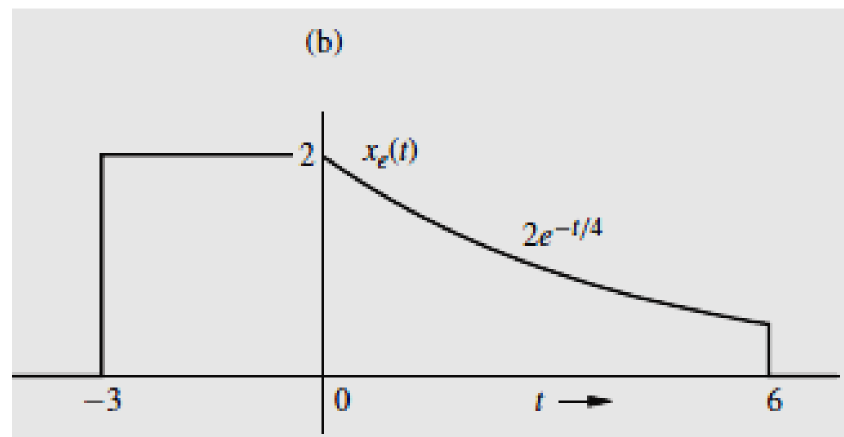
Solution:

$$x(t) = \begin{cases} 2 & -1.5 \le t \le 0 \\ 2e^{-t/2} & 0 \le t \le 3 \\ 0 & otherwise \end{cases}$$



B. Time-expanded by factor 2.

$$x(t/2) = \begin{cases} 2 & -3 \le t \le 0 \\ 2e^{-t/4} & 0 \le t \le 6 \\ 0 & otherwise \end{cases}$$

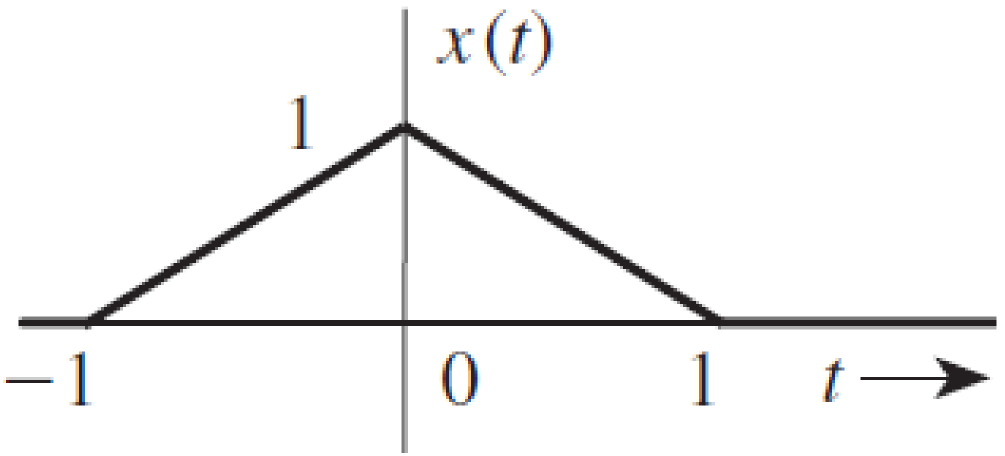


Example:

The signal x(t) is shown in figure. Sketch and describe mathematically this signal:

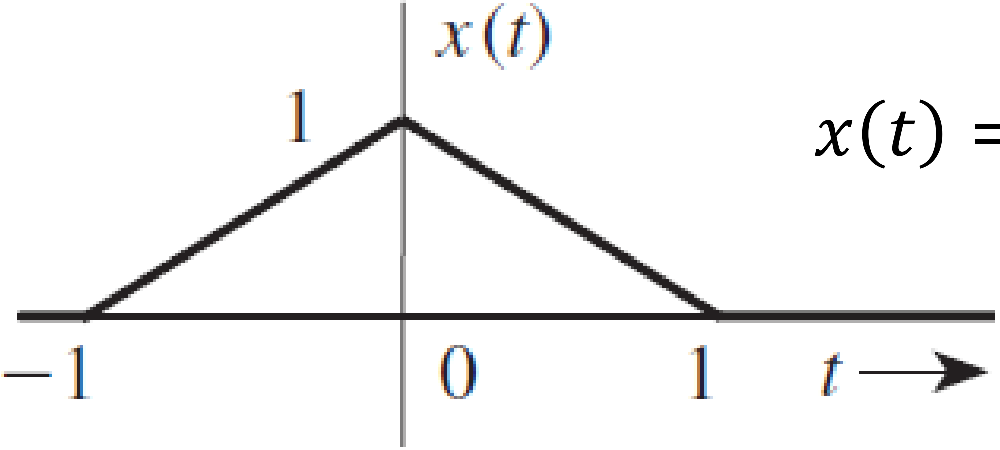
A. Time-compressed by factor 2.

B. Time-expanded by factor 2.



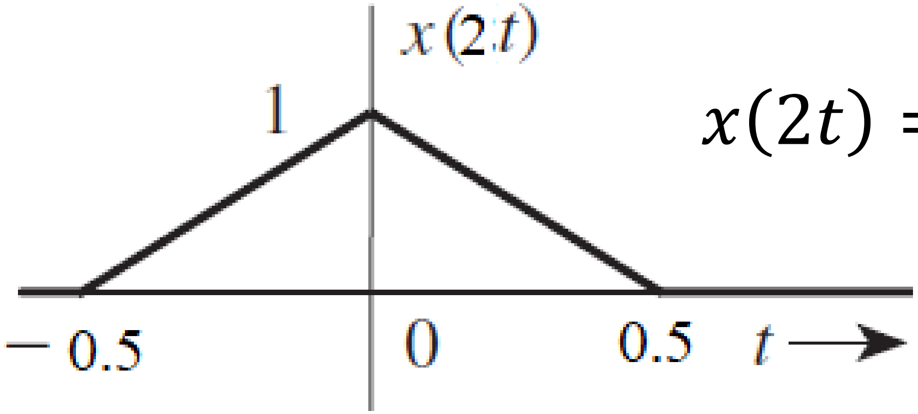
$$x(t) = \begin{cases} 1+t & -1 \le \mathrm{t} \le 0 \\ 1-t & 0 \le \mathrm{t} \le 1 \\ 0 & otherwise \end{cases}$$

Solution:



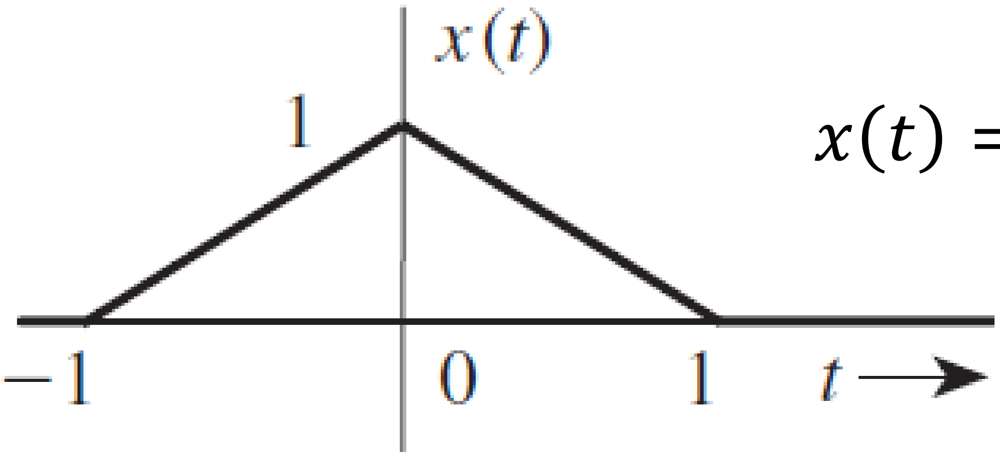
$$x(t) = \begin{cases} 1+t & -1 \le t \le 0 \\ 1-t & 0 \le t \le 1 \\ 0 & otherwise \end{cases}$$

A. Time-compressed by factor 2.



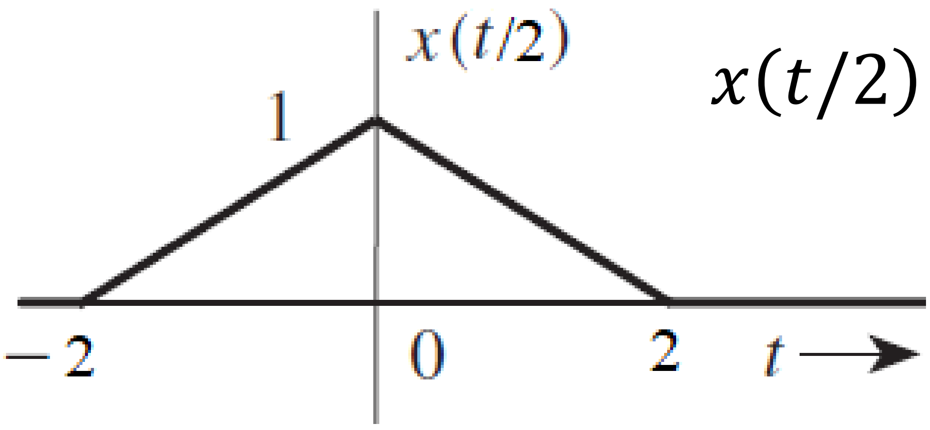
$$x(2t) = \begin{cases} 1+2t & -0.5 \le t \le 0 \\ 1-2t & 0 \le t \le 0.5 \\ 0 & otherwise \end{cases}$$

Solution:



$$x(t) = \begin{cases} 1+t & -1 \le t \le 0 \\ 1-t & 0 \le t \le 1 \\ 0 & otherwise \end{cases}$$

B. Time- expanded by factor 2.



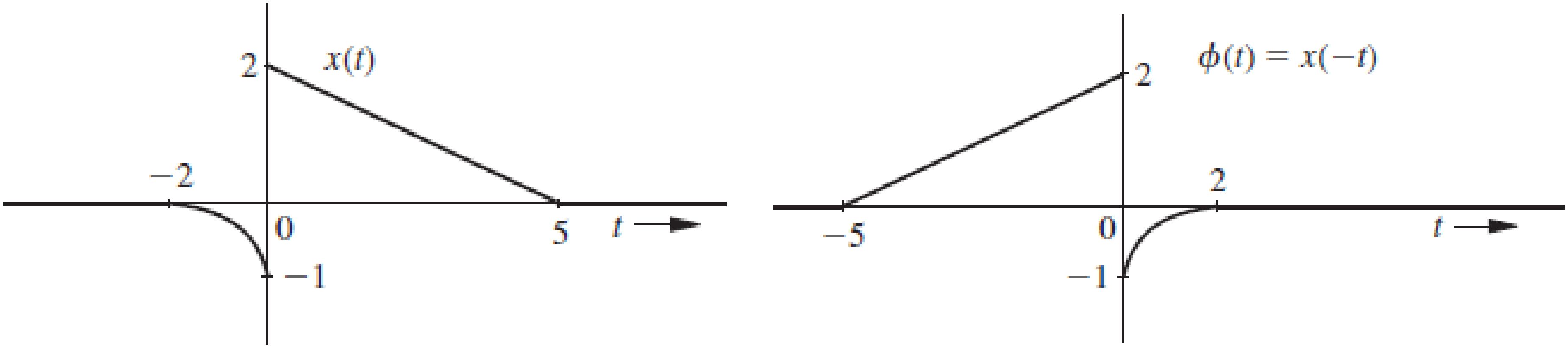
$$x(t/2) = \begin{cases} 1+t/2 & -2 \le t \le 0 \\ 1-t/2 & 0 \le t \le 2 \\ 0 & otherwise \end{cases}$$

SOME USEFUL SIGNAL OPERATIONS

3. Time Reversal:

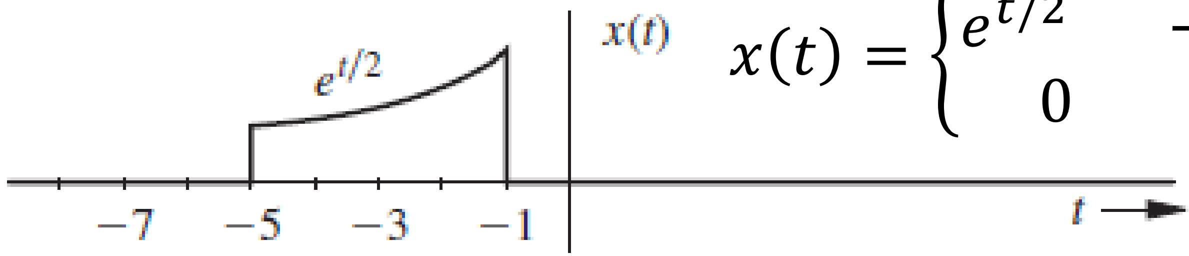
Time reversal is the reflection of x(t) about the vertical axis. The time reversed signal is:

$$\phi(t) = x(-t)$$



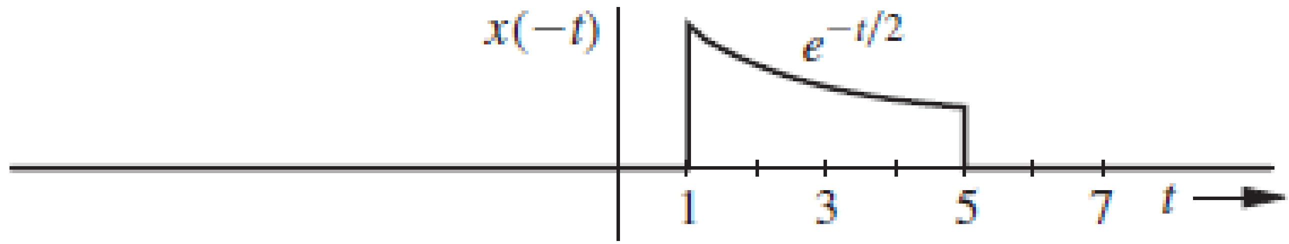
Example:

For the signal x(t) illustrated in figure, sketch x(−t), which is time-reversed x(t).



$$x(t) = \begin{cases} e^{t/2} & -5 \le t \le -1 \\ 0 & otherwise \end{cases}$$

Solution:



$$x(-t) = \begin{cases} e^{-t/2} & 1 \le t \le 5 \\ 0 & otherwise \end{cases}$$

Example:

For the signal x(t) illustrated in figure, sketch:

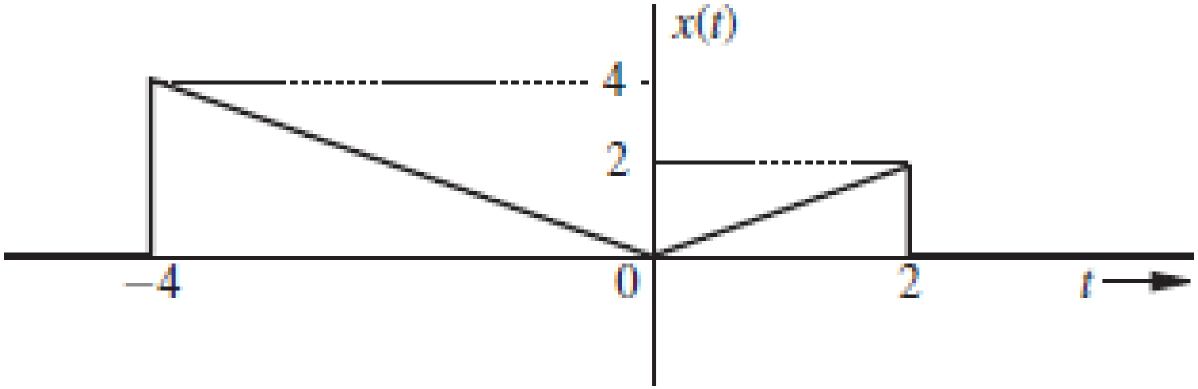
(a) $x(t-4)$

(b) $x(t/1.5)$

(c) $x(-t)$

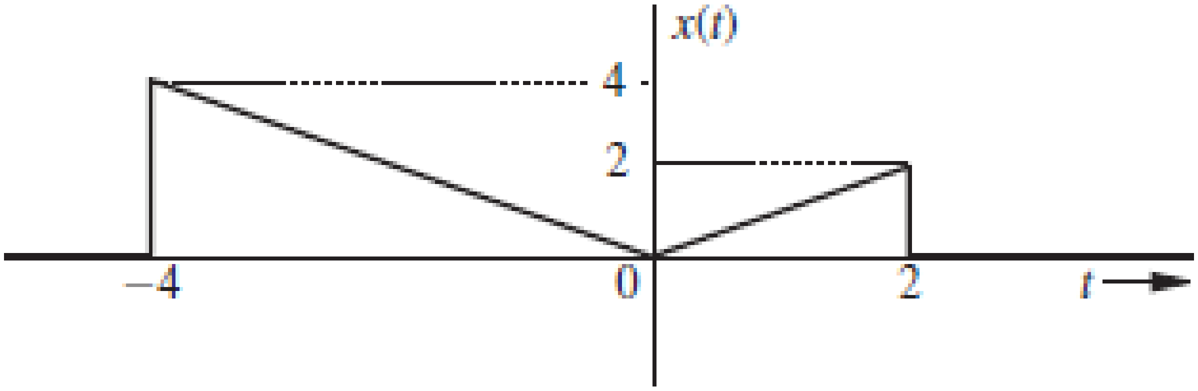
(d) $x(2t-4)$

(e) $x(2-t)$

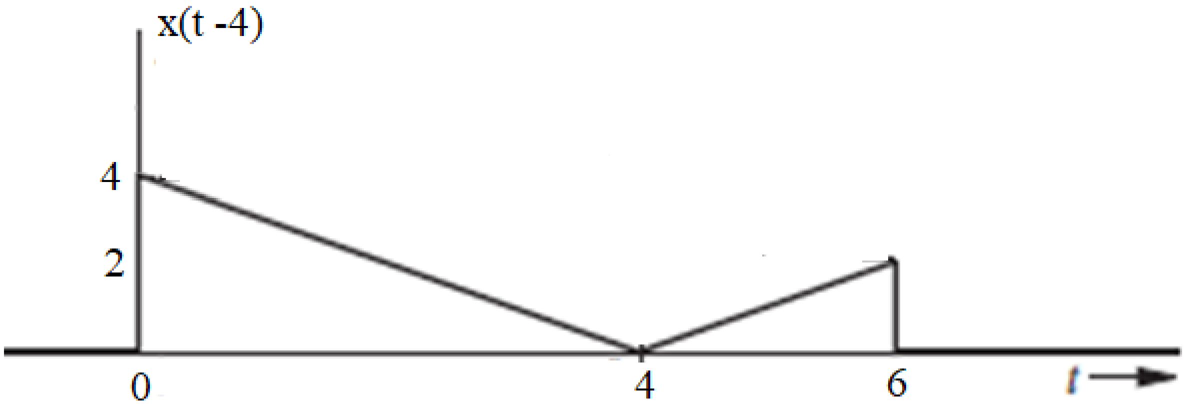


Solution:

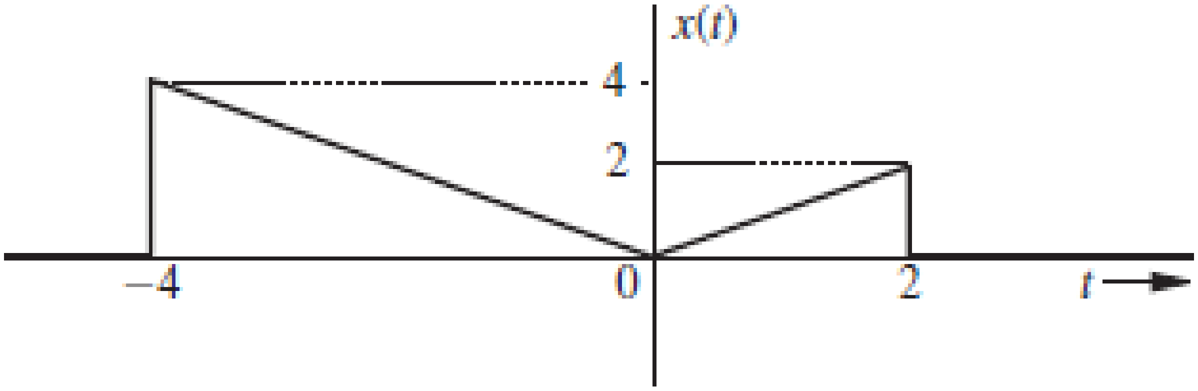
Solution:



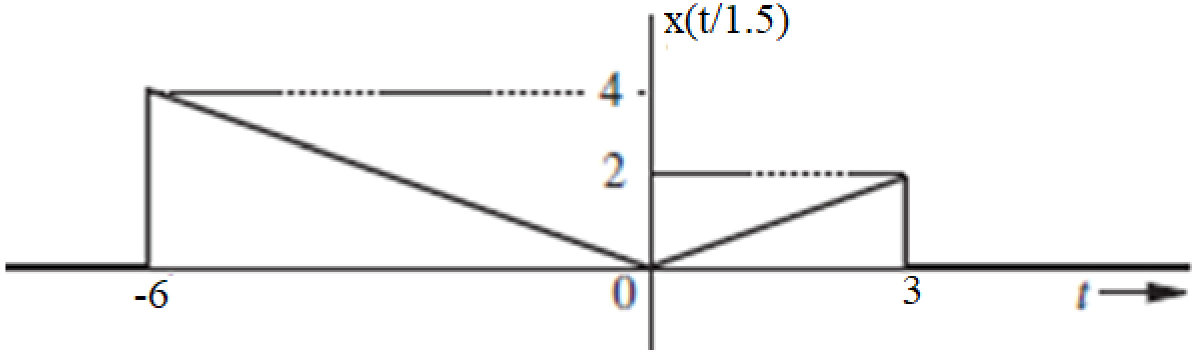
(a) x(t−4)



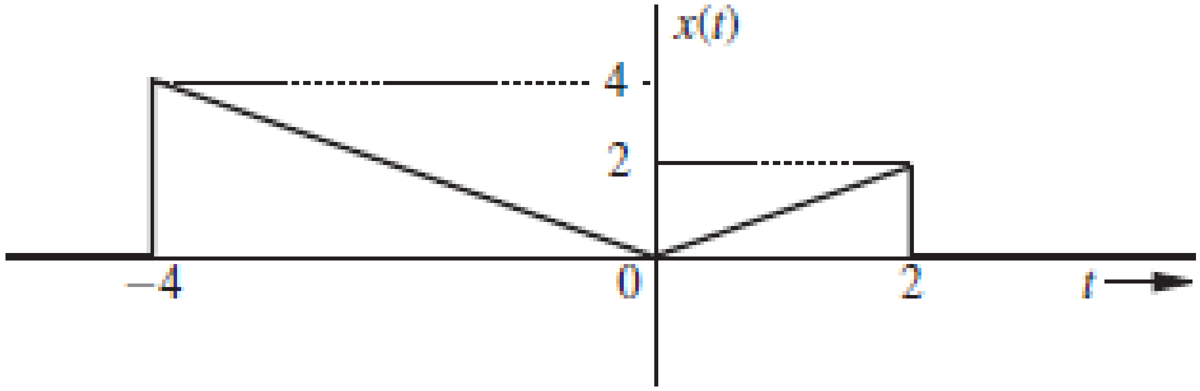
Solution:



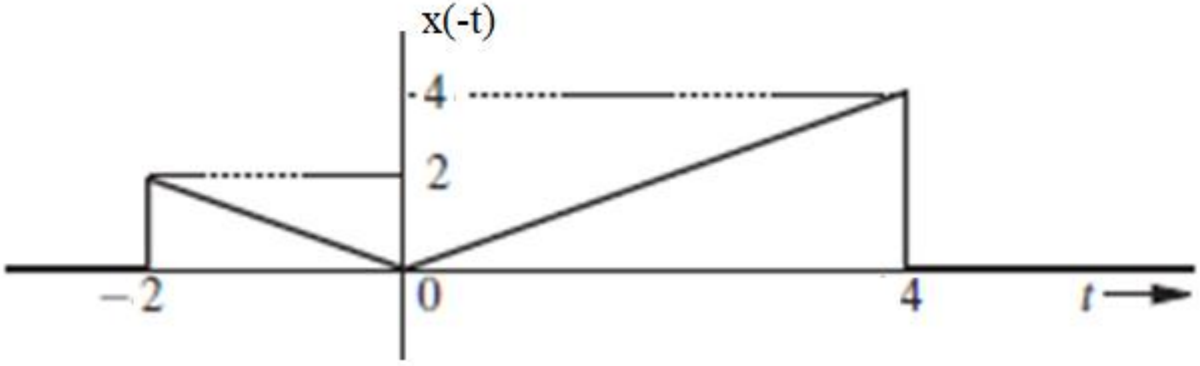
(b) x(t/1.5)



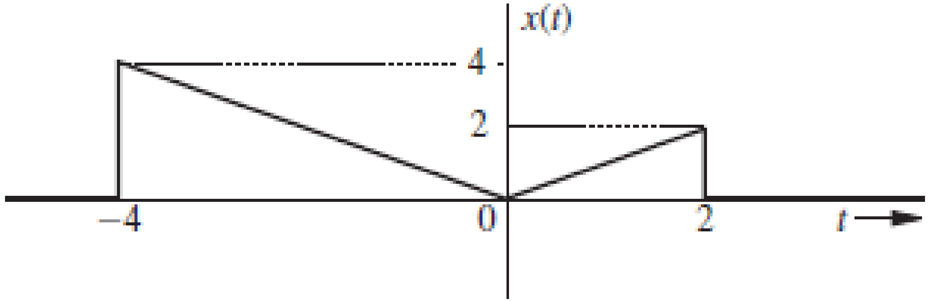
Solution:



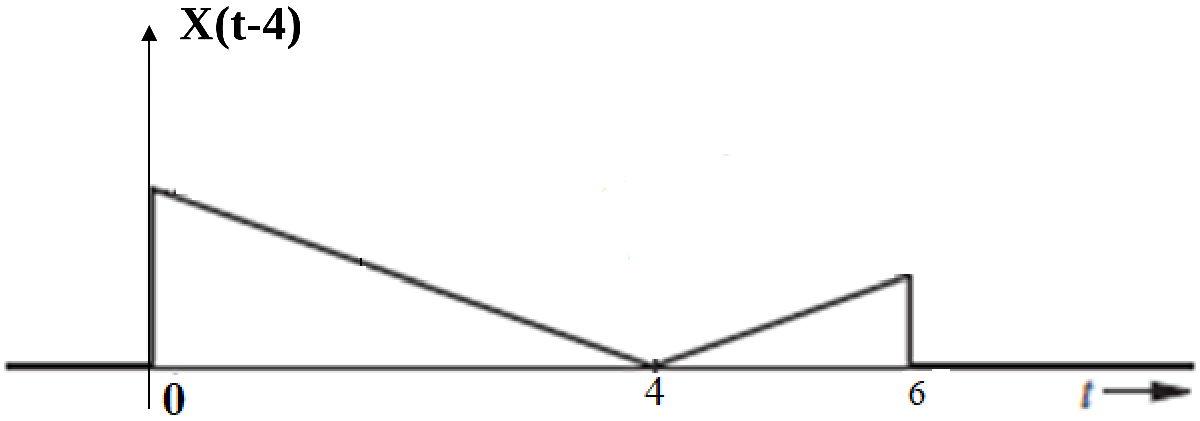
(c) x(-t)

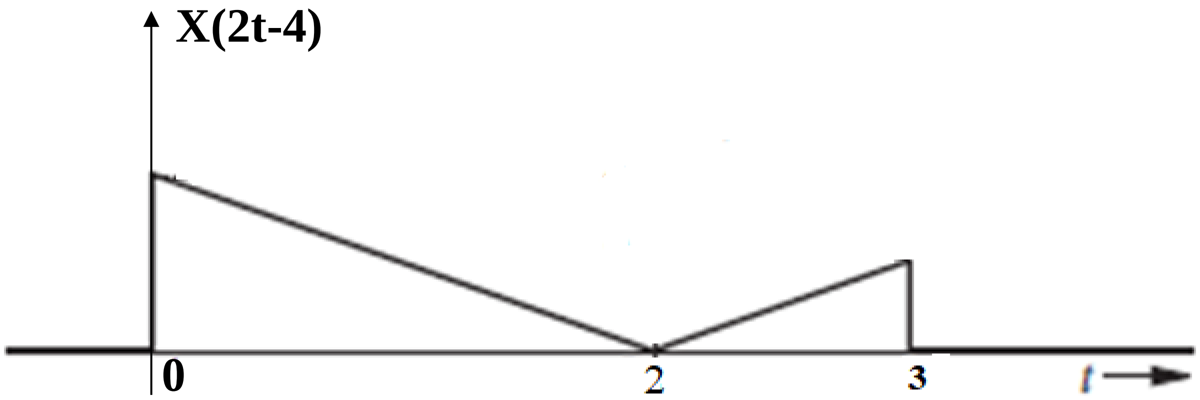


Solution:

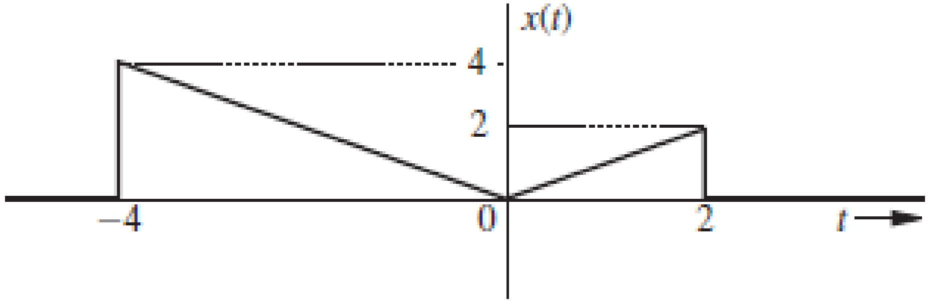


(d) x(2t-4)

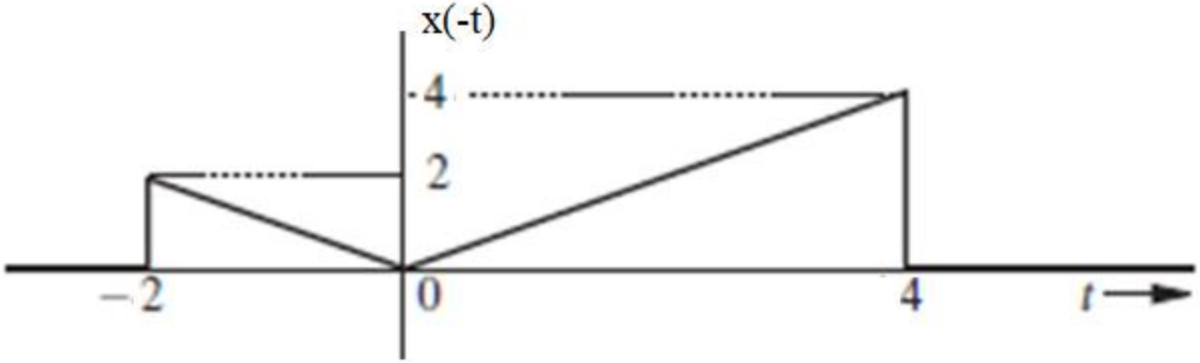


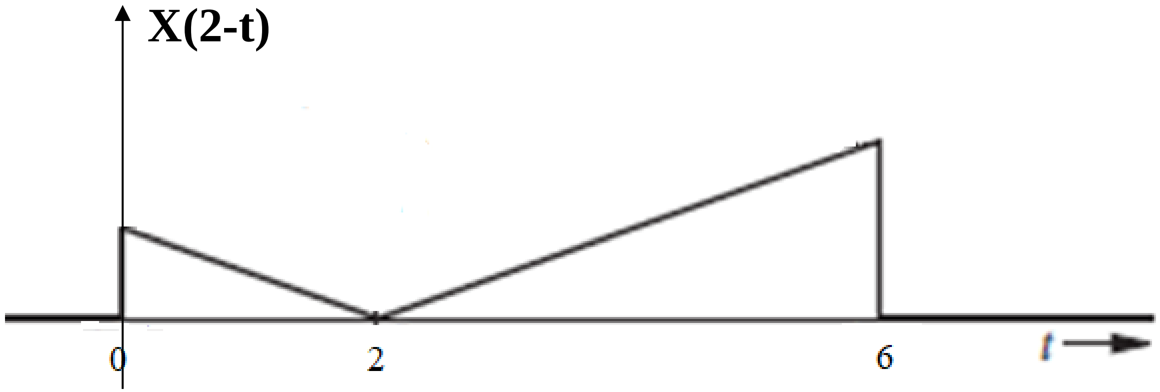


Solution:



(e) x(2−t)





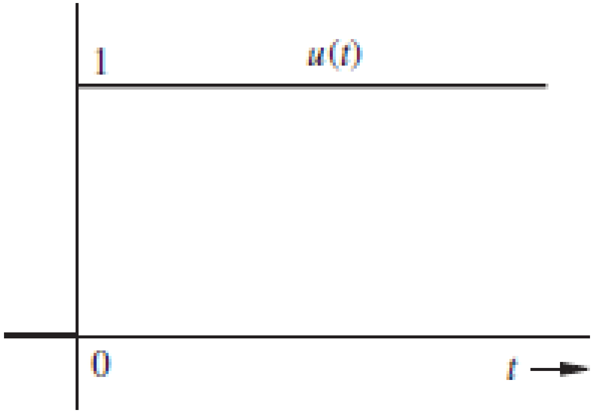
SOME USEFUL SIGNAL MODELS

In the area of signals and systems, **the step, the impulse, and the exponential functions** play very important roles. Not only do they serve as a basis for representing other signals, but their use can simplify many aspects of the signals and systems.

1. The Unit Step Function u(t)

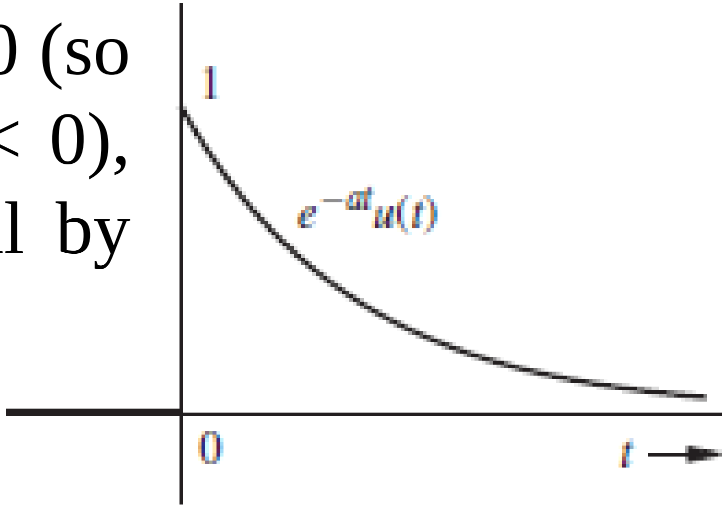
The unit step function u(t) is shown in figure. This function is defined by:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

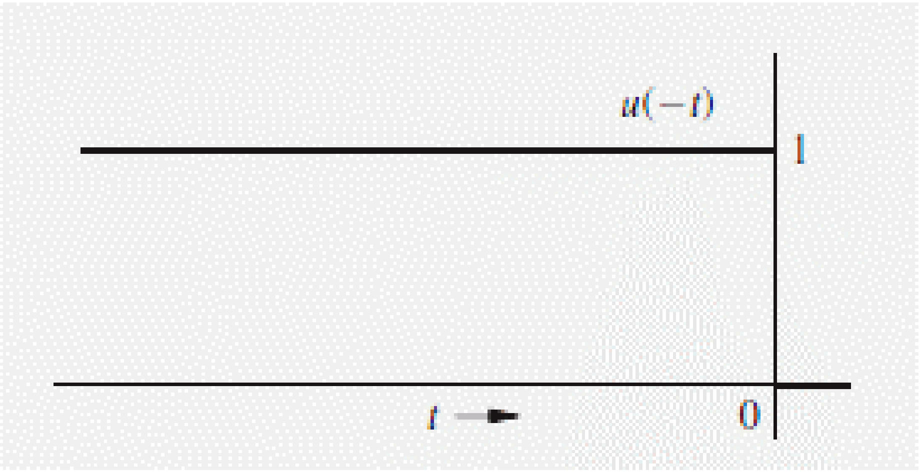


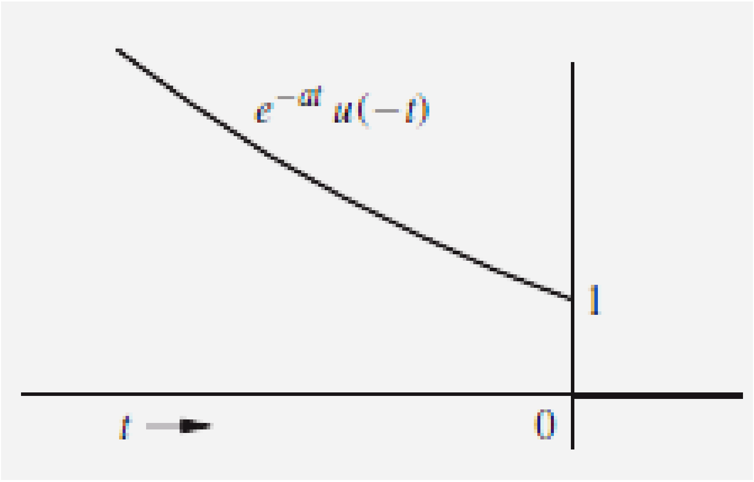
SOME USEFUL SIGNAL MODELS

- If we want a signal to start at t = 0 (so that it has a value of zero for t < 0), we need only multiply the signal by u(t).



- If we want a signal to end at t = 0 (so that it has a value of zero for t > 0), we need only multiply the signal by u(-t).

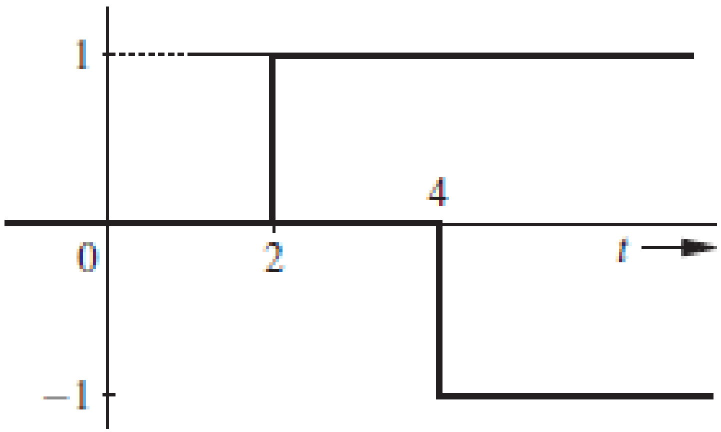


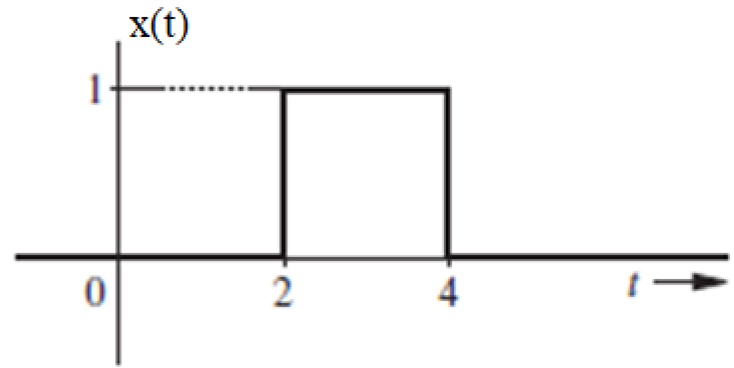


SOME USEFUL SIGNAL MODELS

Representation of the rectangular pulse:

The rectangular pulse *x(t)* can be expressed as the sum of the two delayed unit step functions.

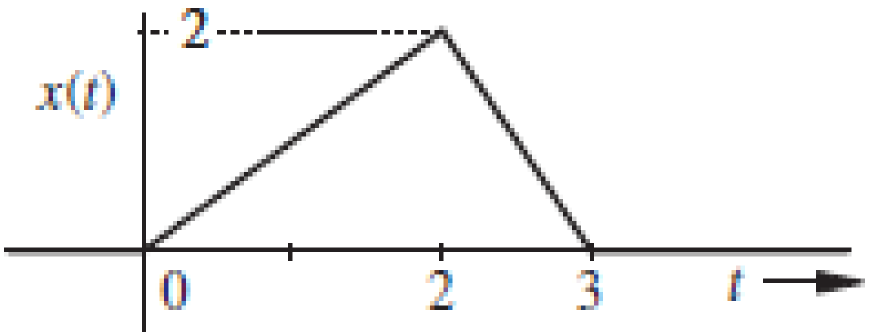




$$x(t) = u(t-2) - u(t-4)$$

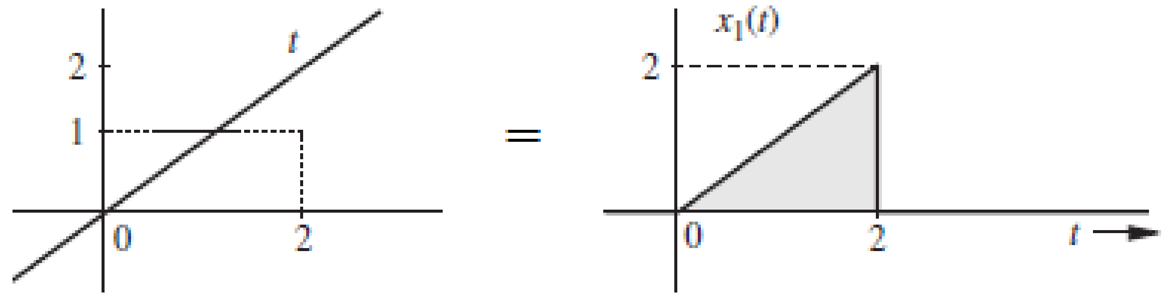
Example:

Use the unit step function to describe the signal x(t) shown in figure.

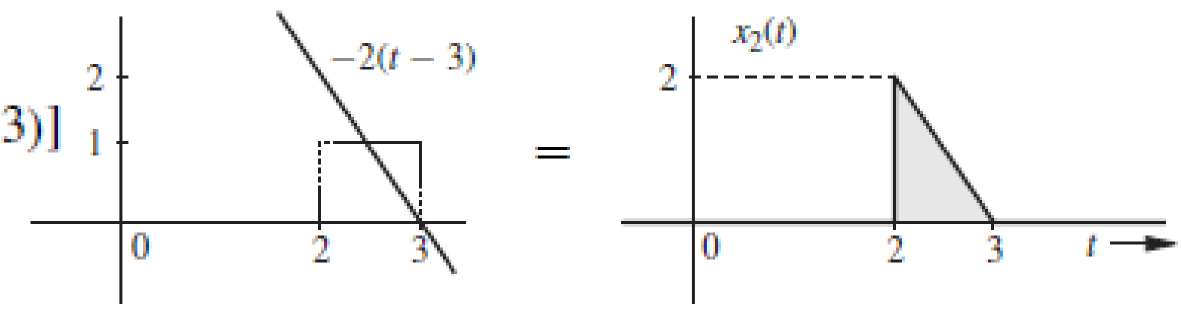


Solution:

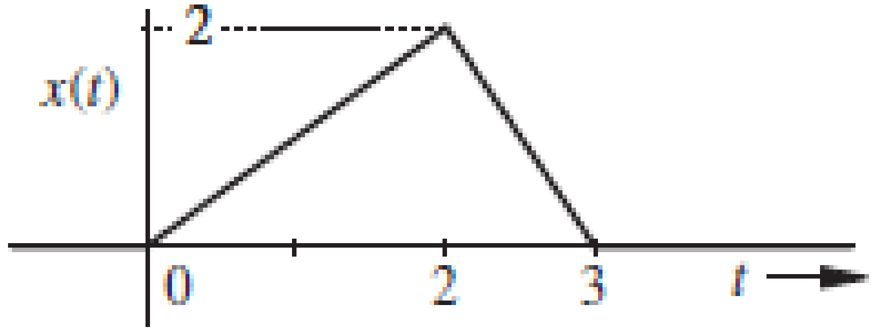
$$x_1(t) = t[u(t) - u(t-2)]$$

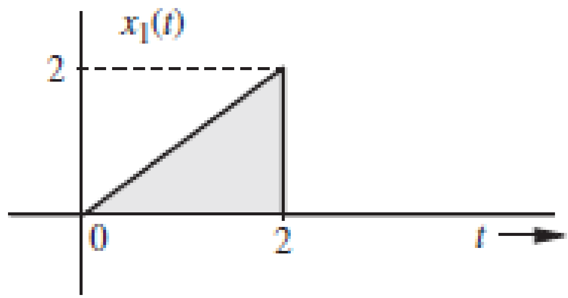


$$x_2(t) = -2(t-3)[u(t-2) - u(t-3)]$$



Solution:





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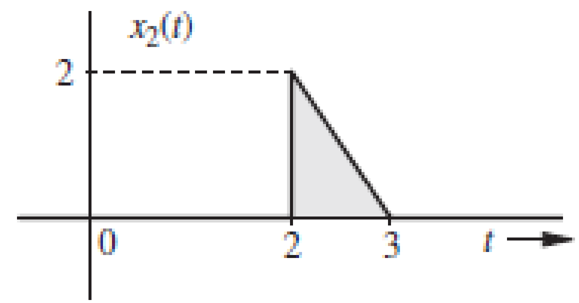


$$x_1(t) = t[u(t) - u(t-2)]$$

$$x_2(t) = -2(t-3)[u(t-2) - u(t-3)]$$

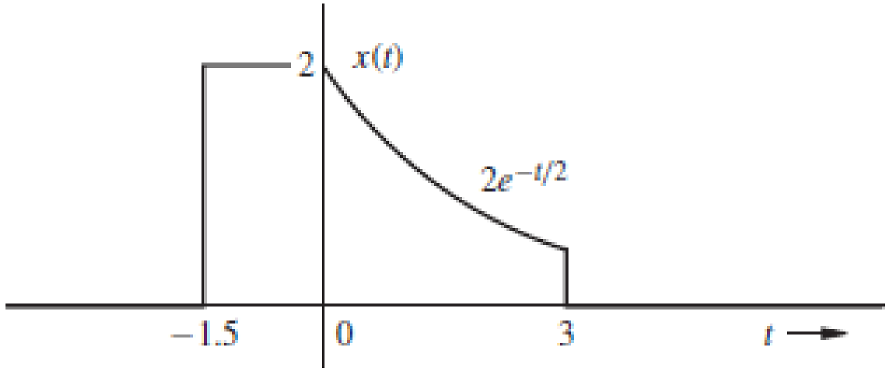
$$x(t) = t[u(t) - u(t-2)] - 2(t-3)[u(t-2) - u(t-3)]$$

$$x(t) = tu(t) - tu(t-2) - 2(t-3)u(t-2) + 2(t-3)u(t-3)$$

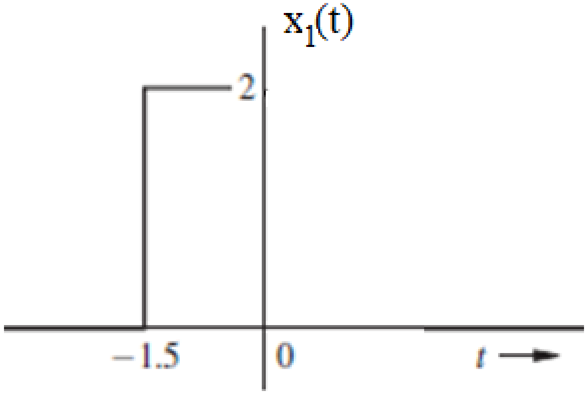
$$x(t) = tu(t) - 3(t-2)u(t-2) + 2(t-3)u(t-3)$$

Example:

Use the unit step function to describe the signal x(t) shown in figure by a single expression valid for all t.



Solution:



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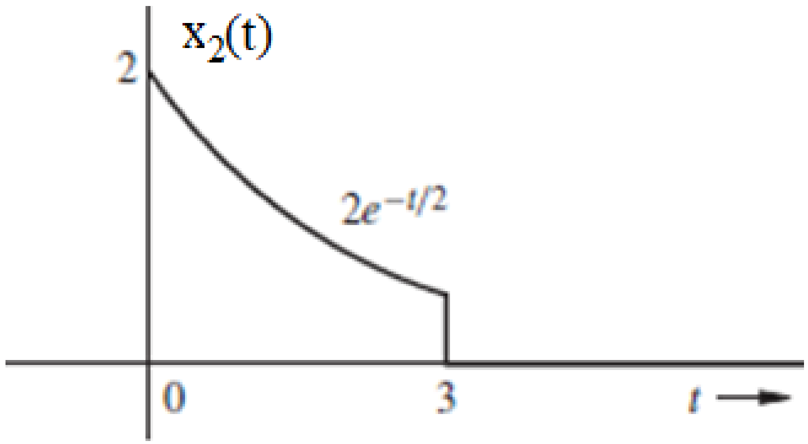
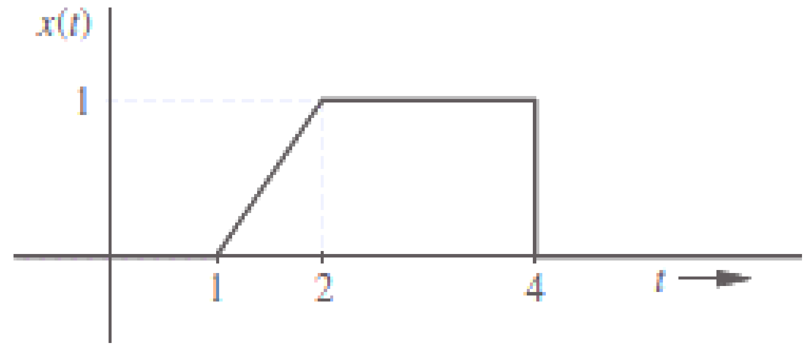


$$x(t) = 2[u(t+1.5) - u(t)] + 2e^{-t/2}[u(t) - u(t-3)]$$

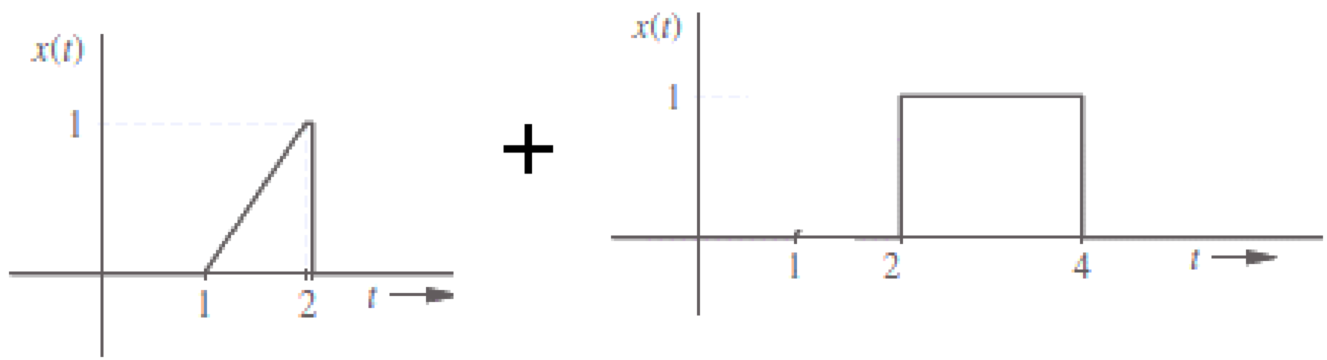
$$x(t) = 2u(t+1.5) - 2u(t) + 2e^{-t/2}u(t) - 2e^{-t/2}u(t-3)$$

Example:

Use the unit step function to describe the signal x(t) shown in figure by a single expression valid for all t.



Solution:



$$x(t) = (t-1)[u(t-1) - u(t-2)] + [u(t-2) - u(t-4)]$$

$$x(t) = (t-1)u(t-1) - (t-1)u(t-2) + u(t-2) - u(t-4)$$

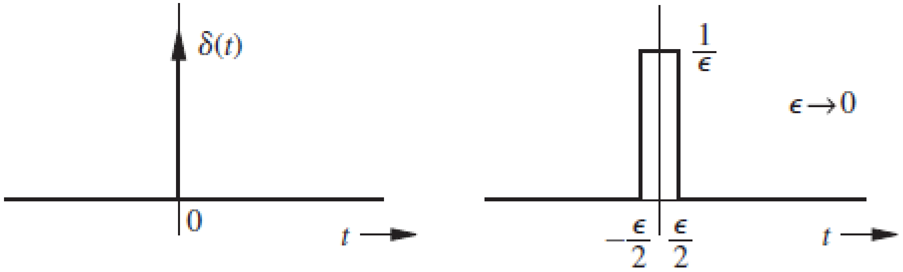
$$x(t) = (t-1)u(t-1) - (t-2)u(t-2) - u(t-4)$$

SOME USEFUL SIGNAL MODELS

2. The Impulse function δ(t)

The unit impulse function δ(t) is one of the most important functions in the study of signals and systems. This function is defined by:

$$\delta(t) = 0 \quad if\ t \neq 0 \qquad\qquad \int_{-\infty}^{\infty} \delta(t)dt = 1$$



The unit impulse can be regarded as a rectangular pulse with a width that has become infinitesimally small, a height that has become infinitely large, and an overall area that has been maintained at unity.

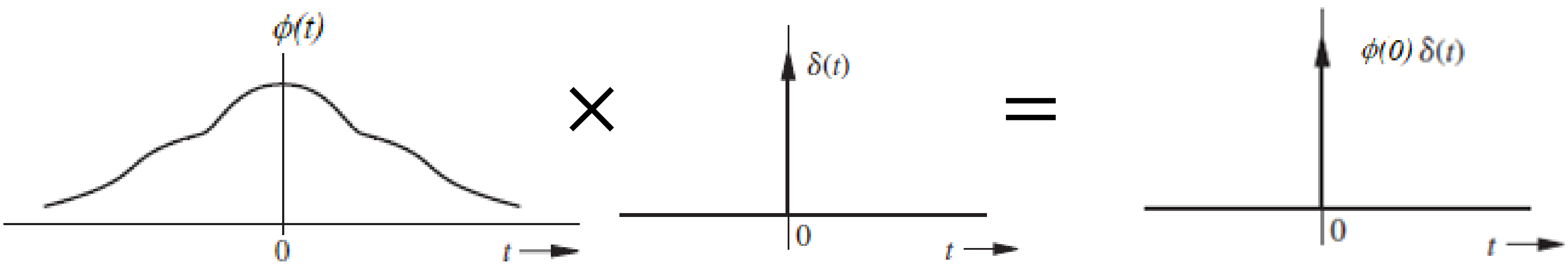
SOME USEFUL SIGNAL MODELS

MULTIPLICATION OF A FUNCTION BY AN IMPULSE

Let us now consider what happens when we multiply the unit impulse δ(t) by a function $\phi(t)$ that is known to be continuous at t = 0.

Since the impulse has nonzero value only at t = 0, and the value of $\phi(t)$ at t = 0 is $\phi(0)$, we obtain

$$\boldsymbol{\phi(t)\delta(t) = \phi(0)\delta(t)}$$

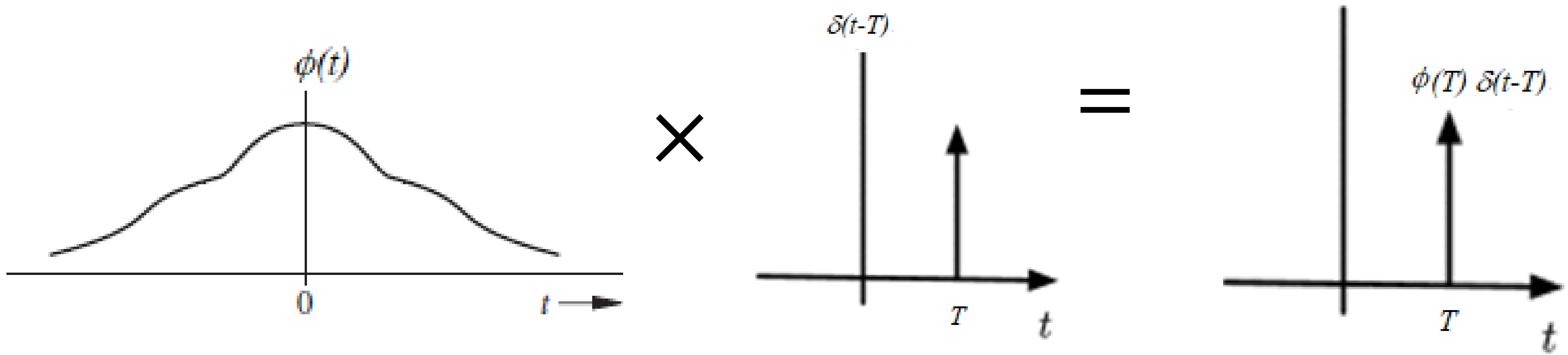


SOME USEFUL SIGNAL MODELS

MULTIPLICATION OF A FUNCTION BY AN IMPULSE

Use of exactly the same argument leads to the generalization of this result, stating that provided $\phi(t)$ is continuous at t = T, $\phi(t)$ multiplied by an impulse δ(t−T) (impulse located at t = T) results in an impulse located at t = T and having strength $\phi(T)$ [the value of φ(t) at the location of the impulse].

$$\boldsymbol{\phi(t)\delta(t-T) = \phi(T)\delta(t-T)}$$



SOME USEFUL SIGNAL MODELS

Notes:

1. $$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

2. $$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)dt = \phi(T)\int_{-\infty}^{\infty} \delta(t)dt = \phi(T)$$

3. $$\int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad \int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

4. $$\frac{du(t)}{dt} = \delta(t)$$

Example:

Show that:

(a) $(t^3+3)\delta(t) = 3\delta(t)$

(b) $\left[\sin\left(t^2 - \frac{\pi}{2}\right)\right]\delta(t) = -\delta(t)$

(c) $e^{-2t}\delta(t) = \delta(t)$

(d) $\frac{\omega^2+1}{\omega^2+9}\delta(\omega-1) = \frac{1}{5}\delta(\omega-1)$

Solution:

$$\boldsymbol{\phi(t)\delta(t-T) = \phi(T)\delta(t-T)}$$

(a) $$(t^3+3)\delta(t) = (0^3+3)\delta(t) = 3\delta(t)$$

(b) $$\left[\sin\left(t^2 - \frac{\pi}{2}\right)\right]\delta(t) = \left[\sin\left(0^2 - \frac{\pi}{2}\right)\right]\delta(t)$$

$$= \left[\sin\left(-\frac{\pi}{2}\right)\right]\delta(t) = -\delta(t)$$

Solution:

(c)

$$e^{-2t}\delta(t) = e^{-2\times 0}\delta(t) = \delta(t)$$

(d)

$$\frac{\omega^2 + 1}{\omega^2 + 9}\delta(\omega - 1) = \frac{1^2 + 1}{1^2 + 9}\delta(\omega - 1)$$

$$= \frac{2}{10}\delta(\omega - 1)$$

$$= \frac{1}{5}\delta(\omega - 1)$$

Example:

Show that:

(a) $\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt = 1$

(b) $\int_{-\infty}^{\infty} \delta(t-2) \cos\left(\frac{\pi t}{4}\right) dt = 0$

(c) $\int_{-\infty}^{\infty} e^{-2(x-t)} \delta(2-t)\, dt = e^{-2(x-2)}$

Solution:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)dt = \phi(T)$$

(a) $$\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = e^{-j\omega \times 0} = 1$$

(b) $$\int_{-\infty}^{\infty} \delta(t-2) \cos\left(\frac{\pi t}{4}\right) dt = \cos\left(\frac{\pi \times 2}{4}\right) = 0$$

Solution:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)dt = \phi(T)$$

(c)

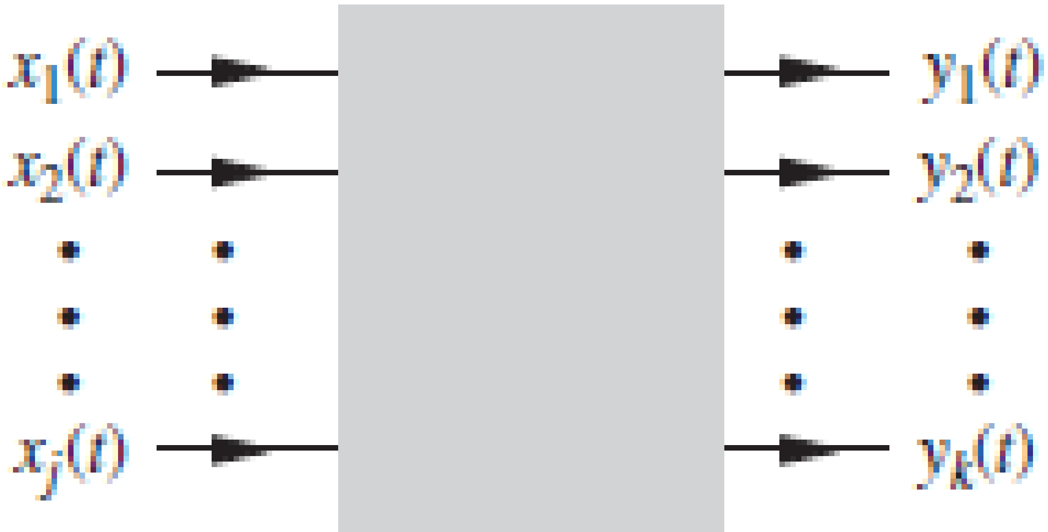
$$\int_{-\infty}^{\infty} e^{-2(x-t)}\delta(2-t)dt = e^{-2(x-2)}$$

Systems

- systems are used to process signals to allow modification or extraction of additional information from the signals.
- A system may consist of physical components (hardware realization) or of an algorithm that computes the output signal from the input signal (software realization).
- A physical system consists of interconnected components, which are characterized by their terminal (input–output) relationships. In addition, a system is governed by laws of interconnection. For example, in electrical systems, the terminal relationships are the familiar voltage-current relationships for the resistors, capacitors, inductors, transformers, transistors, and so on, as well as the laws of interconnection (i.e., Kirchhoff's laws). We use these laws to derive mathematical equations relating the outputs to the inputs. These equations then represent a mathematical model of the system.

Systems

A system can be conveniently illustrated by a "black box" with input variables $x_1(t)$, $x_2(t)$, . . ., $x_j(t)$ and output variables $y_1(t)$, $y_2(t)$,. . ., $y_k(t)$ as shown in figure.



CLASSIFICATION OF SYSTEMS

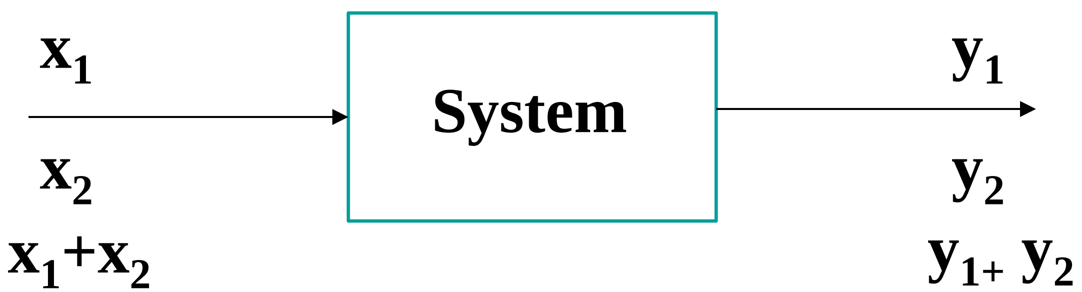
Systems may be classified broadly in the following categories:

1. Linear and nonlinear systems
2. Time-invariant and time-variant systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and noncausal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Stable and unstable systems

CLASSIFICATION OF SYSTEMS

1. Linear and nonlinear systems:

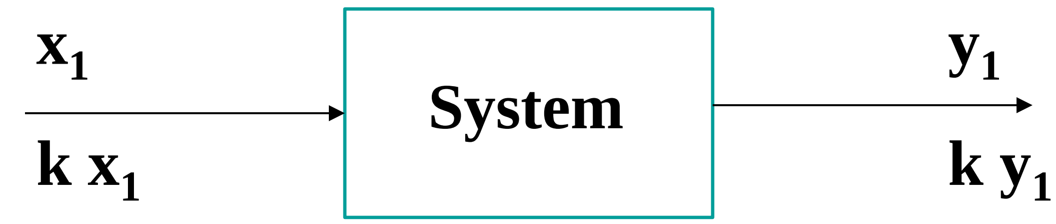
- A system whose output is proportional to its input is an example of a linear system. But linearity implies more than this; it also implies the additivity property.
- This property may be expressed as follows: for a linear system, if an input x_1 acting alone has an effect y_1, and if another input x_2, also acting alone, has an effect y_2, then, with both inputs acting on the system, the total effect will be $y_1 + y_2$.

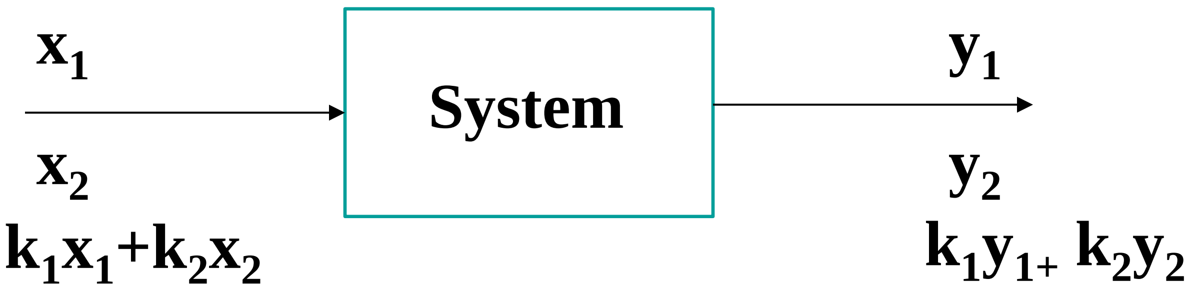


CLASSIFICATION OF SYSTEMS

1. Linear and nonlinear systems:

In addition, a linear system must satisfy the homogeneity or scaling property, which states that for arbitrary real or imaginary number k, if an input is increased k-fold, the effect also increases k-fold.





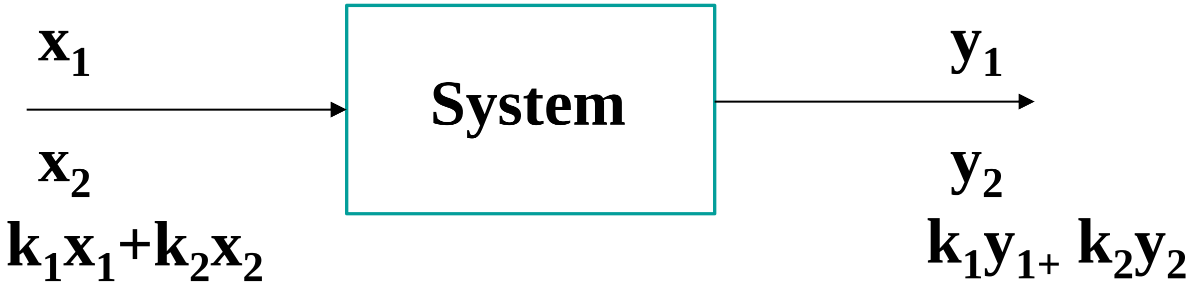
Example:

Show that the system described by the following differential equation is linear

$$\frac{dy(t)}{dt} + 3y(t) = x(t)$$

Where x(t) is the system i/p signal and y(t) is its o/p signal.

Solution:



Let $x(t)= x_1(t)$, and $y(t) =y_1(t)$.

$$\frac{dy_1(t)}{dt} + 3y_1(t) = x_1(t) \quad \text{------} \quad (1)$$

Solution:

Let $x(t) = x_2(t)$, and $y(t) = y_2(t)$.

$$\frac{dy_2(t)}{dt} + 3y_2(t) = x_2(t) \quad \text{- · - · -} \quad (2)$$

Multiply equation (1) by k1, and equation (2) by k2 and added equation (1) and (2):

$$k_1 \frac{dy_1(t)}{dt} + 3k_1 y_1(t) = k_1 x_1(t)$$

$$k_2 \frac{dy_2(t)}{dt} + 3k_2 y_2(t) = k_2 x_2(t)$$

$$k_1 \frac{dy_1(t)}{dt} + k_2 \frac{dy_2(t)}{dt} + 3k_1 y_1(t) + 3k_2 y_2(t) = k_1 x_1(t) + k_2 x_2(t) \quad \text{- · - · -} \quad (3)$$

Solution:

$$k_1 \frac{dy_1(t)}{dt} + k_2 \frac{dy_2(t)}{dt} + 3k_1 y_1(t) + 3k_2 y_2(t) = k_1 x_1(t) + k_2 x_2(t)$$

$$\frac{d\big(k_1 y_1(t) + k_2 y_2(t)\big)}{dt} + 3\big(k_1 y_1(t) + k_2 y_2(t)\big) = k_1 x_1(t) + k_2 x_2(t)$$

This is the system equation with the i/p $k_1 x_1(t) + k_2 x_2(t)$, and the o/p $k_1 y_1(t) + k_2 y_2(t)$, so the system is linear.

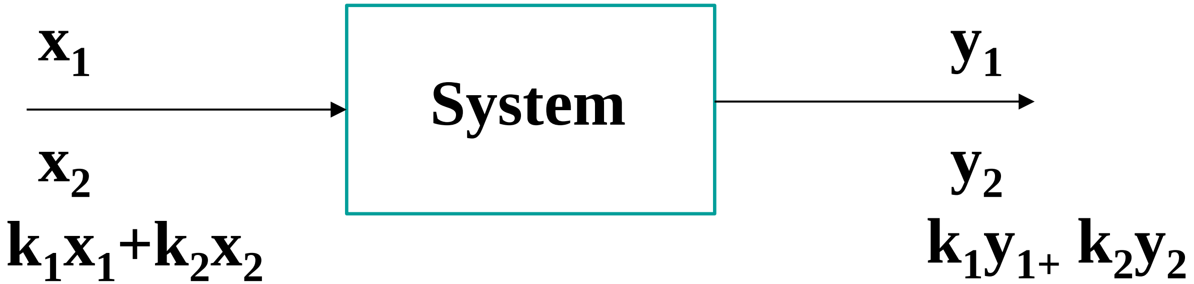
Example:

Show that the system described by the following differential equation is nonlinear

$$y(t)\frac{dy(t)}{dt} + 3y(t) = x(t)$$

Where x(t) is the system i/p signal and y(t) is its o/p signal.

Solution:



Let x(t)= x_1(t), and y(t) =y_1(t).

$$y_1(t)\frac{dy_1(t)}{dt} + 3y_1(t) = x_1(t) \quad \text{------} \quad (1)$$

Solution:

Let $x(t) = x_2(t)$, and $y(t) = y_2(t)$.

$$y_2(t)\frac{dy_2(t)}{dt} + 3y_2(t) = x_2(t) \quad \text{- · - · - ·} \quad (2)$$

Multiply equation (1) by k_1, and equation (2) by k_2 and added equation (1) and (2):

$$k_1 y_1(t)\frac{dy_1(t)}{dt} + 3k_1 y_1(t) = k_1 x_1(t)$$

$$k_2 y_2(t)\frac{dy_2(t)}{dt} + 3k_2 y_2(t) = k_2 x_2(t)$$

$$k_1 y_1(t)\frac{dy_1(t)}{dt} + k_2 y_2(t)\frac{dy_2(t)}{dt} + 3k_1 y_1(t) + 3k_2 y_2(t) = k_1 x_1(t) + k_2 x_2(t)$$

$$\text{- · - · - ·} \quad (3)$$

Solution:

Let $x(t) = k_1x_1(t)+k_2x_2(t)$, and $y(t) = k_1y_1(t)+ k_2y_2(t)$.

$$y(t)\frac{dy(t)}{dt} + 3y(t) = x(t)$$

$$(k_1y_1(t)+ k_2y_2(t))\frac{d\big(k_1y_1(t) + k_2y_2(t)\big)}{dt} + 3\big(k_1y_1(t) + k_2y_2(t)\big)$$
$$= k_1x_1(t) + k_2x_2(t)$$

$$k_1y_1(t)\frac{d\big(k_1y_1(t)\big)}{dt} + k_1y_1(t)\frac{d\big(k_2y_2(t)\big)}{dt} + k_2y_2(t)\frac{d\big(k_1y_1(t)\big)}{dt}$$
$$+k_2y_2(t)\frac{d\big(k_2y_2(t)\big)}{dt} + 3\big(k_1y_1(t) + k_2y_2(t)\big) = k_1x_1(t) + k_2x_2(t)$$

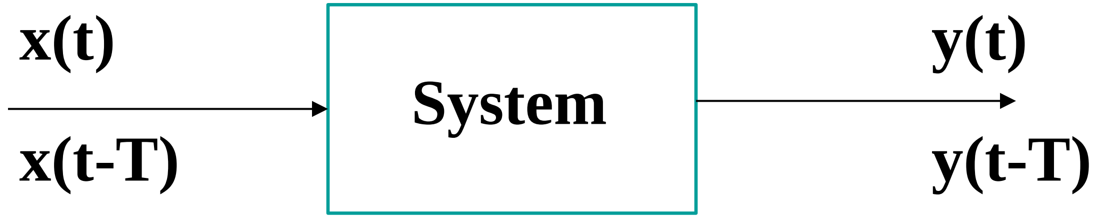
4

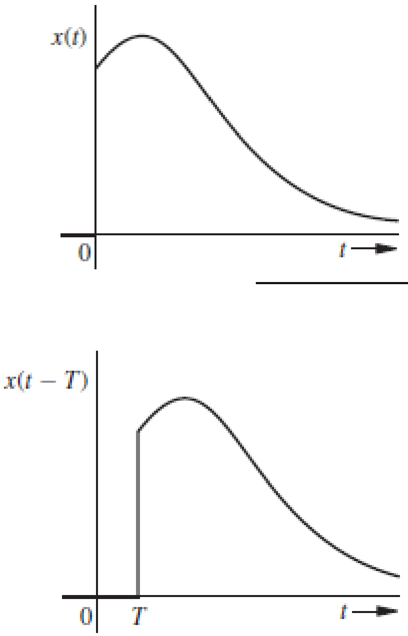
Equation 3, and 4 are not the same so the system is nonlinear.

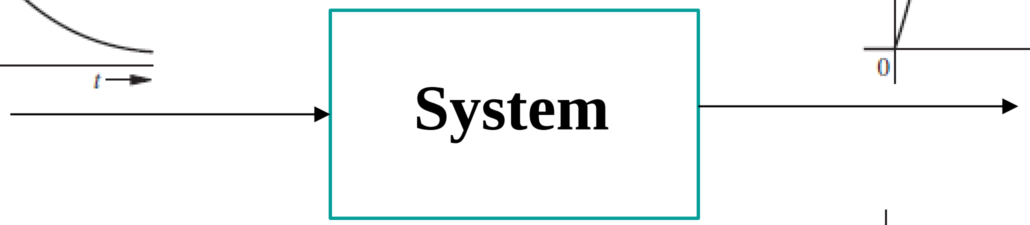
CLASSIFICATION OF SYSTEMS

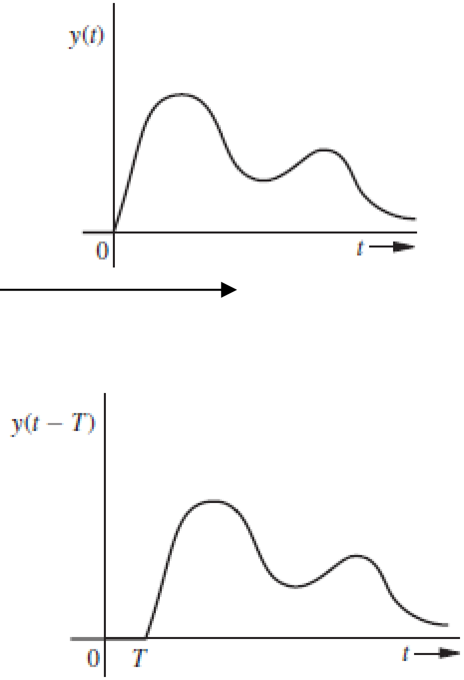
2. Time-invariant and time-variant systems

For time invariant systems, if the input is delayed by T seconds, the output is the same as before but delayed by T seconds.









CLASSIFICATION OF SYSTEMS

2. Time-invariant and time-variant systems

A system with an input–output relationship described by a linear differential equation of the form shown below is a **linear time-invariant** (LTI) system when the coefficients **a_i and b_i** of such equation are **constants**. If these coefficients are functions of time, then the system is a linear time-varying system.

$$a_0 \frac{d^N y(t)}{dt^N} + a_1 \frac{d^{N-1} y(t)}{dt^{N-1}} + \cdots + a_N y(t) = b_{N-M} \frac{d^M x(t)}{dt^M} + \cdots + b_{N-1} \frac{dx(t)}{dt} + b_N x(t)$$

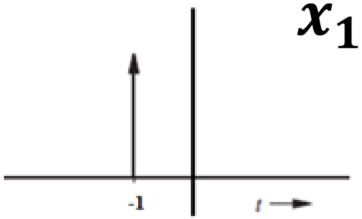
Example:

Determine the time invariance of the following system:

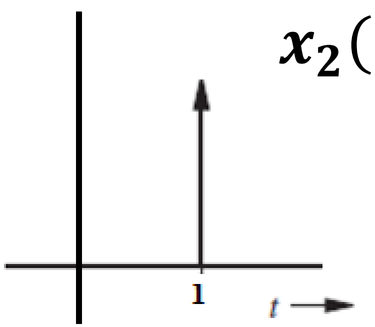
$$y(t) = x(t)u(t)$$

Where x(t) is the system i/p signal and y(t) is its o/p signal.

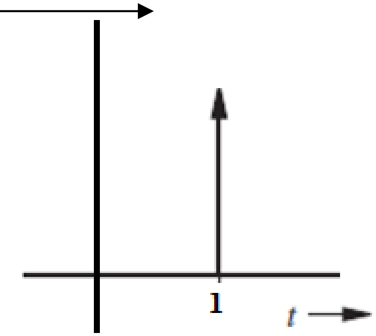
Solution:



$$\boldsymbol{x_1(t) = \delta(t+1)}$$ → **System** → $$\boldsymbol{y_1(t) = 0}$$



$$\boldsymbol{x_2(t) = \delta(t-1)}$$ → **System** → $$\boldsymbol{y_2(t) = \delta(t-1)}$$



The system is time varying.

CLASSIFICATION OF SYSTEMS

3. Instantaneous and dynamic systems

- A system is said to be instantaneous (or memoryless) if its output at any instant t depends, at most, on the strength of its input(s) at the same instant t, and not on any past or future values of the input(s). Otherwise, the system is said to be dynamic (or a system with memory).
- In resistive networks, for example, any output of the network at some instant t depends only on the input at the instant t. In these systems, past history is irrelevant in determining the response.
- A system whose response at t is completely determined by the input signals over the past T seconds [interval from $(t-T)$ to t] is a finite-memory system with a memory of T seconds.
- Networks containing inductive and capacitive elements generally have infinite memory because the response of such networks at any instant t is determined by their inputs over the entire past $(-\infty, t)$.

Example:

Determine whether the following systems are memoryless or dynamic:

$$a. y(t-1) = 2x(t-1)$$

$$b. y(t) = (t-1)x(t)$$

$$c. y(t) = \frac{dx(t)}{dt}$$

Where x(t) is the system i/p signal and y(t) is its o/p signal.

Solution:

$$a. y(t-1) = 2x(t-1)$$

The output at time t−1 is just twice the input at the same time t−1. Since the output at a particular time depends only on the strength of the input at the same time, the system is **memoryless**.

Solution: $b. \ y(t) = (t-1)x(t)$

The output y(t) at time t is just the input x(t) at the same time t multiplied by the (time-dependent) coefficient t −1. Since the output at a particular time depends only on the strength of the input at the same time, the system is **memoryless**.

$$c. \ y(t) = \frac{dx(t)}{dt}$$

$$y(t) = \lim_{T \to \infty} \frac{x(t) - x(t-T)}{T}$$

Since the output at a particular time depends on more than just the input at the same time, the system is **not memoryless (dynamic)**.

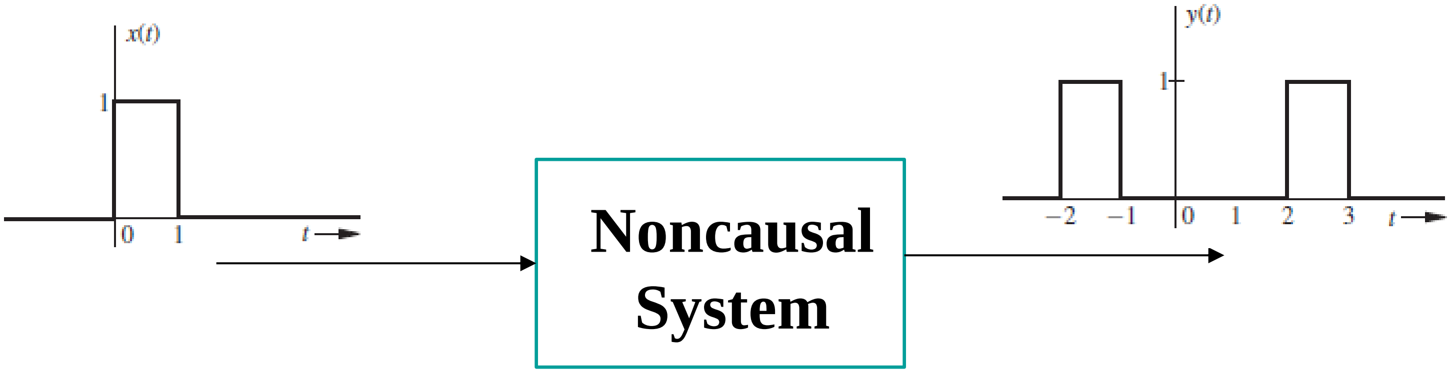
CLASSIFICATION OF SYSTEMS

4. Causal and noncausal systems

- A causal system is one for which the output at any instant t_0 depends only on the value of the input x(t) for $t \leq t_0$.
- In other words, the value of the output at the present instant depends only on the past and present values of the input x(t), not on its future values.
- To put it simply, in a causal system the output cannot start before the input is applied.
- If the response starts before the input, it means that the system knows the input in the future and acts on this knowledge before the input is applied. A system that violates the condition of causality is called a noncausal system.

CLASSIFICATION OF SYSTEMS

4. Causal and noncausal systems



Example:

Determine whether the following systems are causal:

$$a.\, y(t) = x(t+1)$$

$$b.\, y(t+1) = x(t)$$

Where x(t) is the system i/p signal and y(t) is its o/p signal.

Solution:

$$a.\, y(t) = x(t+1)$$

The output at time t depends on the input at future time of t +1. Clearly the system is **not causal**.

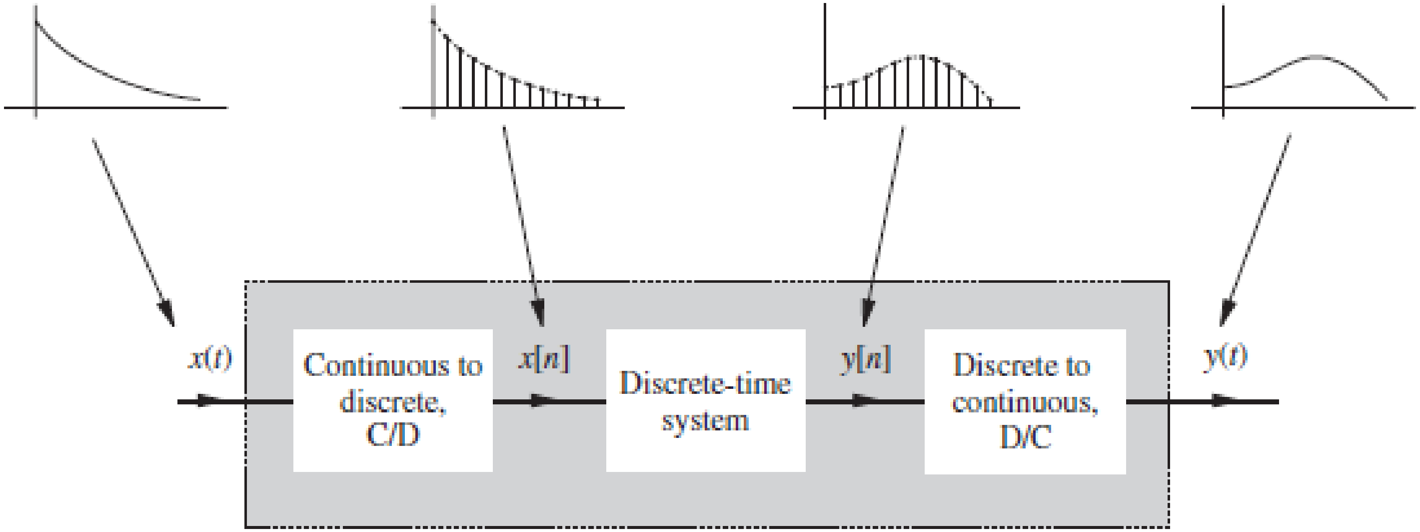
$$b.\, y(t+1) = x(t)$$

the output at time t +1 depends on the input one second in the past, at time t. Since the output does not depend on future values of the input, the system is **causal**.

CLASSIFICATION OF SYSTEMS

5. Continuous-time and discrete-time systems

Systems whose inputs and outputs are continuous-time signals are continuous-time systems. Systems whose inputs and outputs are discrete-time signals are discrete-time systems.



CLASSIFICATION OF SYSTEMS

6. Analog and digital systems

A system whose input and output signals are analog is an analog system; a system whose input and output signals are digital is a digital system.

7. Stable and unstable systems

Systems can also be classified as stable or unstable systems. If every bounded input applied at the input terminal results in a bounded output, the system is said to be stable in the BIBO (bounded-input/bounded-output) sense.