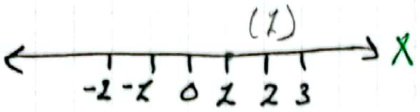
CALCULUS

3 - space

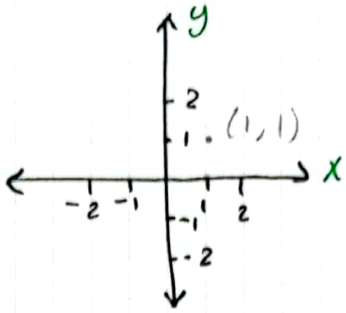
18/10/2023

wednesday

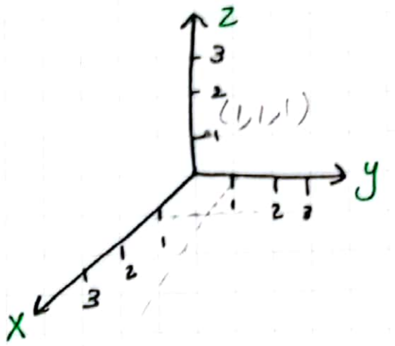
1 - space



2 - space

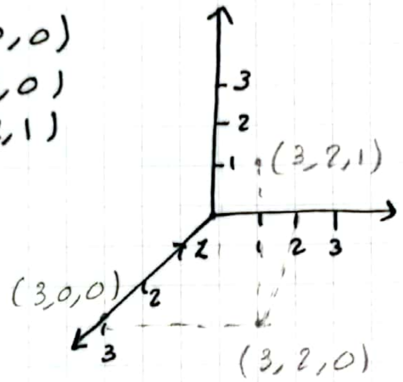


3 - space



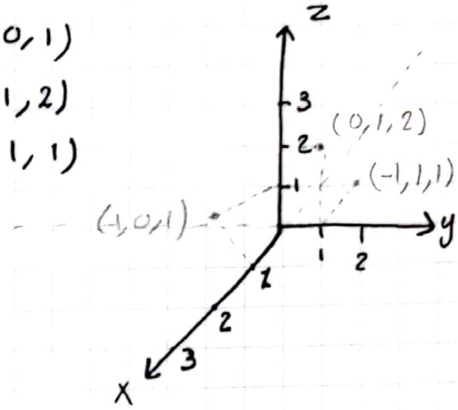
EX: sketch the points in 3-space :-

① (3,0,0)
② (3,2,0)
③ (3,2,1)



EX: sketch the points :

① (1,0,1)
② (0,1,2)
③ (-1,1,1)



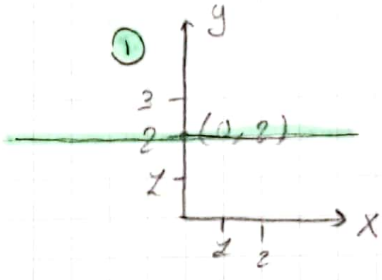
EX: sketch the points :

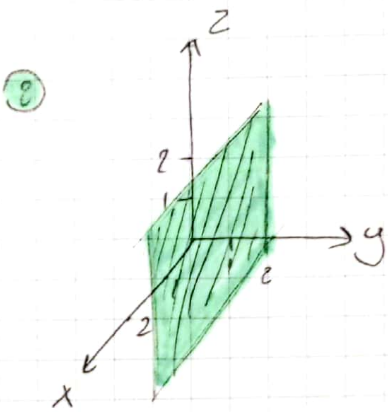
1 (y=2) in 2-space

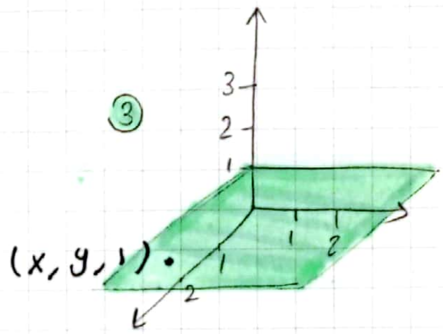
2 (y=2) in 3-space

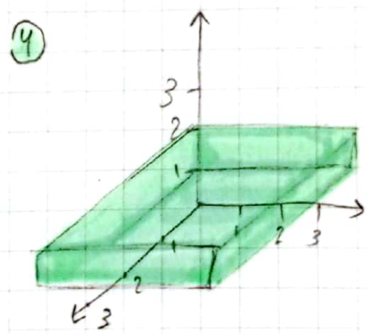
3 (z=1) in 3-space

4 1 ≤ z ≤ 2 in 3-sp









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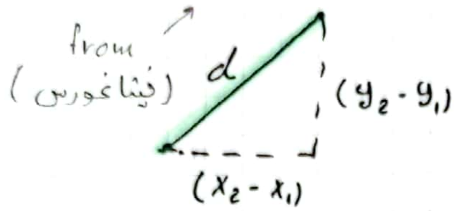
Distance between Two points:

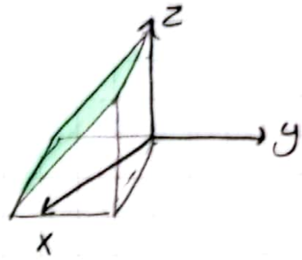
① In 2 dimentions

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$



*② In 3-space

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$



Midpoint:

① 2-space

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$$

② 3-space

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right), \left(\frac{z_1 + z_2}{2}\right)$$

Find the distance between:

(3, 4, -1) AND (2, 6, 5)

$d = \sqrt{1 + 4 + 36}$

$d = \sqrt{41}$

Find Midpoint:-

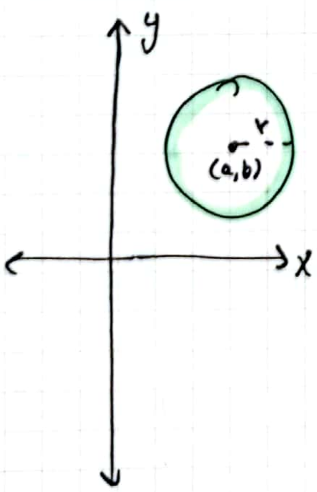
(3, 4, -1) AND (2, 6, 5)

$M = (2.5, 5, 2)$

Sphere

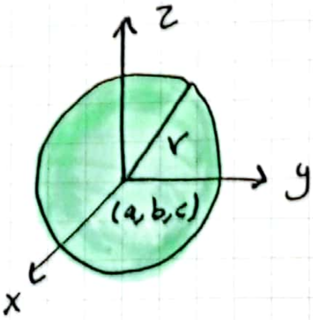
Circle → 2-space

$$(x-a)^2 + (y-b^2) = r^2 \Rightarrow x^2 + y^2 = 1 \text{ (unit circle)}$$



- equation of sphere in 3-sp:

$$(x-a)^2 + (y-b^2) + (z-c)^2 = r^2$$



CALCULUS III
FOLLOWER

18/10/2023
WED

EX: Find the center and radius for the following sphere :-

1. $(x+1)^2+(y-2)^2+(z-4)^2=25 \Rightarrow r=5 \ , \ c=(-1,2,4)$
2. $(x+7)^2+(y-\pi)^2+(z-e)^2=11 \Rightarrow r=\sqrt{11} \ , \ c=(-7,\pi,e)$

EX: Find the equation of the sphere with A and B endpoints to its diameter.

$A=(2,-3,+1)$
$B=(4,1,1)$

$\Rightarrow$ general equation $=(x-a)^2+(y-b)^2+(z-c)^2=r^2$
we need To find r and c.

① r : $d=\sqrt{(4-2)^2+(1--3)^2+(1-1)^2}$
$=\sqrt{20}/2 \Rightarrow \sqrt{5}$
$r=\sqrt{5}$

② c : $\left(\frac{2+4}{2}\right),\left(\frac{1+-3}{2}\right),\left(\frac{1+1}{2}\right)$
$c:(3,-1,1)$

③ equation $=(x-3)^2+(y+1)^2+(z-1)^2=5$

END OF LECTURE ⟷

FOLLOWER

23/10/2023
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- $(x-a)^2+(y-b)^2+(z-c)^2=r^2 \Leftarrow$ sphere
- $(x-a)^2+(y-b)^2+(z-c)^2=k$
  - $\rightarrow k>0$ sphere
  - $\rightarrow k=0$ point
  - $\rightarrow k\leq 0$ No graph

Describe the surface :

$x^2+y^2+z^2+10x+4y+2z-19=0$ ← لإكمال المربع يجب ان يكون معامل $x^2=1$.
$x^2+y^2+z^2+10x+4y+2z=19$
$(x^2+10x)+(y^2+4y)+(z^2+2z)=19$
$(x^2+10x+25-25)+(y^2+4y+4-4)+(z^2+2z+1-1)$

$10x \Rightarrow \frac{10}{2}=(5)^2=25$

$(x^2+10x+25)+(y^2+4y+4)+(z^2+2z+1)=49$
$(x+5)^2+(y+2)^2+(z+1)^2=49$

$4y \Rightarrow \frac{4}{2}=(2)^2=4$

$\Rightarrow c=(-5,-2,-1) \quad r=\sqrt{49}=7$ #sphere

$2z \Rightarrow \frac{2}{2}=(1)^2=1$

CALCULUS III

Cylindrical surfaces

23/10/2023

MON

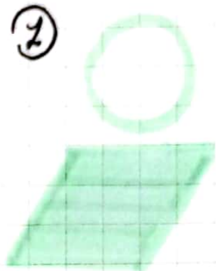
■ Describe the sphere :-

$$x^2+y^2+z^2+2x-2y+2z+3=0$$
$$(x^2+2x)+(y^2-2y)+(z^2+2z)+3=0$$
$$(x^2+2x+1)+(y^2-2y+1)+(z^2+2z+1)-3+3=0$$
$$(x+1)^2+(y-1)^2+(z+1)^2=0 \Rightarrow \text{point } \#$$

1 EX: For the sphere $(x-1)^2+(y+2)^2+(z-3)^2=5$
Find the intersection of the sphere with xy- plane :- $(z=0)$

$$(z=0) \Rightarrow (x-1)^2+(y+2)^2+(0-3)^2=5$$
$$(x-1)^2+(y+2)^2+9=5$$
$$(x-1)^2+(y+2)^2=-4 \quad \leftarrow \text{No intersection}$$

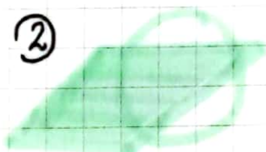
1



2 EX: For the previous EX, Find the intersection for $z=3$.

$$(z=3) \Rightarrow (x-1)^2+(y+2)^2+(0)^2=5$$
$$C=(1,-2) \quad r=\sqrt{5} \quad \# \text{ circle } \#$$

2



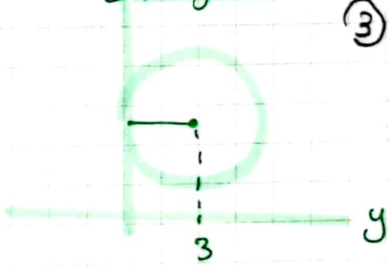
3 Find the equation of the sphere :
$C=(2,-3,6)$ and tangent at xz-plane.

$$\Rightarrow (x-2)^2+(y+3)^2+(z-6)^2=r^2$$
$$(x-2)^2+9+(z-6)^2=r^2$$
$$\searrow (r=+3) \text{ from } y=0 .$$

$\Rightarrow$ If the tangent was at xy-plane then
$r=6$.

Z Tangent

3

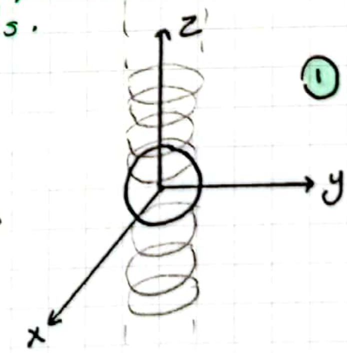


Cylindrical Surfaces

■ equation in 3 dimention with two variablels. $\nearrow$ variables.

EX: $x^2+y^2=1$ in 3 - space #1 abt z-axis

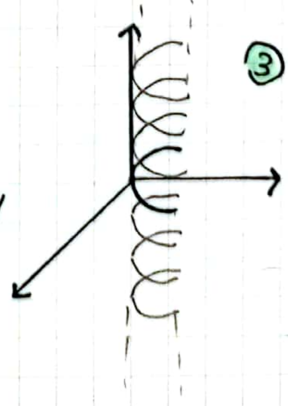
1



EX: $x^2+z^2=1$ In 3-space # 2 abt y-axis

EX: $y=x^2$ in 3-Space # 3

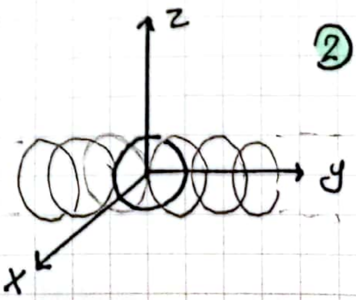
3



EX: $y=\sin x$ In 3 -Space #4

4

2



CALCULUS III

11.2 / VECTORS

25/10/2023

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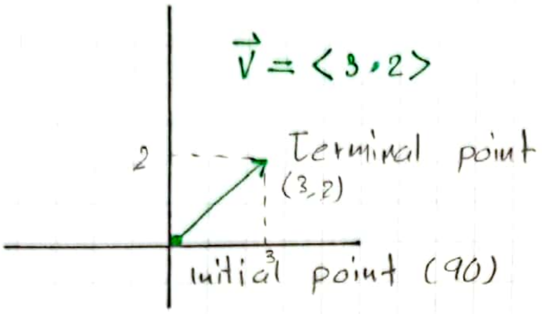
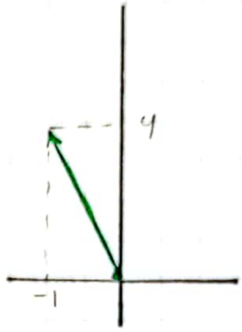
- **VECTOR** → Magnitude
  → Direction

$\vec{V} = \langle 3, 2 \rangle$



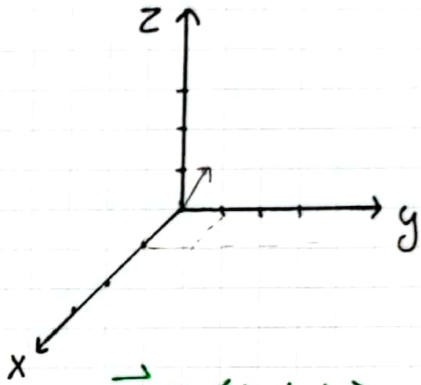
- $\vec{w} = \langle -1, 4 \rangle$ in 2-space



⟨vector⟩ , (Point)

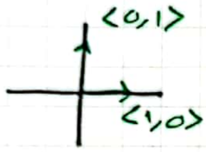
In 3-space:

$\vec{U} = \langle a, b, c \rangle$

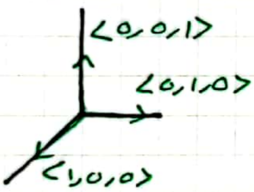


$\vec{U} = \langle 1, 1, 1 \rangle$

- **Remark**



$\hat{i} = \langle 1, 0 \rangle$
2-space
$\hat{j} = \langle 0, 1 \rangle$



$\hat{i} = \langle 1, 0, 0 \rangle$
$\hat{j} = \langle 0, 1, 0 \rangle$
3-space
$\hat{k} = \langle 0, 0, 1 \rangle$

- **Applications on vectors:**

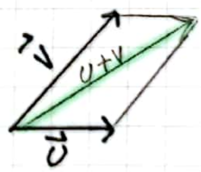
\# Rules: Let $\vec{U}, \vec{V}$ be Two vectors:

$\left.\begin{array}{l}\vec{U} = \langle U_1, U_2 \rangle \\ \vec{V} = \langle V_1, V_2 \rangle\end{array}\right] \to 2D$ $\quad \left.\begin{array}{l}\vec{U} = \langle U_1, U_2, U_3 \rangle \\ \vec{V} = \langle V_1, V_2, V_3 \rangle\end{array}\right] \to 3\text{-}D$

① $\vec{U} + \vec{V} \Rightarrow \langle U_1 + V_1, U_2 + V_2 \rangle$ or $\langle U_1 + V_1, U_2 + V_2, U_3 + V_3 \rangle$

EX: $\vec{U} = \langle 3, 4 \rangle$
$\vec{V} = \langle 2, 5 \rangle$

$\vec{U} + \vec{V} = \langle 5, 9 \rangle$



② $\vec{U} + \vec{V} \Rightarrow \vec{V} + \vec{U}$ (ulternative)

③ $K\vec{U} \Rightarrow \langle KU_1, KU_2, KU_3 \rangle$

EX: $\vec{U} = \langle 1, 2, 3 \rangle$ Find $4\vec{U} = \langle 4, 8, 12 \rangle$

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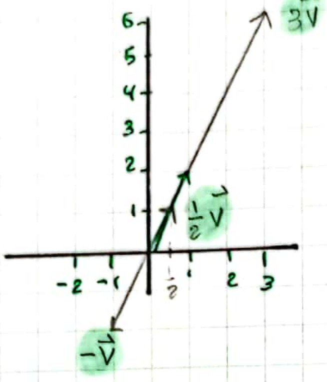
Rules :

EX: $\vec{v} = \langle 1, 2 \rangle$ Find:

1. $3\vec{v} = \langle 3, 6 \rangle$
2. $\frac{1}{2}\vec{v} = \langle \frac{1}{2}, 1 \rangle$
3. $-\vec{v} = \langle -1, -2 \rangle$



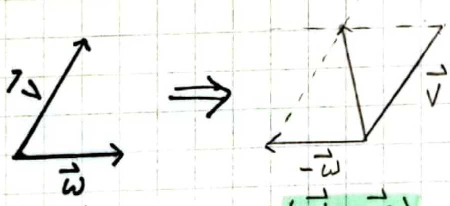
Rules

4. $\vec{0}$ $\langle 0, 0 \rangle$ 2-space
   $\langle 0, 0, 0 \rangle$ 3-space

$\vec{U} + \vec{0} = \vec{U}$
$\vec{U} + (-\vec{U}) = \vec{0}$.

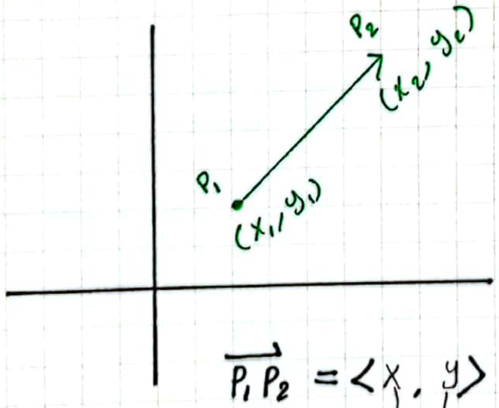
Rules

5. $\vec{v} - \vec{\omega} \Rightarrow \vec{v} + (-\vec{\omega})$



$(\vec{v} + \vec{\omega})$ $(\vec{v} - \vec{\omega})$

Remark



$\overrightarrow{P_1P_2} = \langle x, y \rangle$

$x_2 - x_1$ $y_2 - y_1$

So That it starts at $\langle 0, 0 \rangle$.

EX: Find $5\vec{\omega} - \vec{v}$:

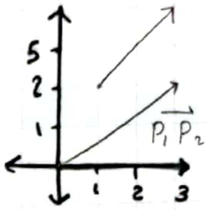
$\vec{\omega} = \langle -7, -2, 5 \rangle$

$\vec{v} = \langle 0, 2, 4 \rangle$

1. $5\vec{\omega} \Rightarrow \langle -35, -10, 25 \rangle$
2. $\langle -35, -10, 25 \rangle - \langle 0, 2, 4 \rangle$

$= \langle -35, -12, 21 \rangle$

EX: write the vector $\overrightarrow{P_1P_2}$ where: $P_1 = (1, 2)$ $P_2 = (3, 5)$



$\overrightarrow{P_1P_2} = \langle 2, 3 \rangle$ from origin.

Norm of the vector

(Length) $\Rightarrow \|\vec{v}\|$

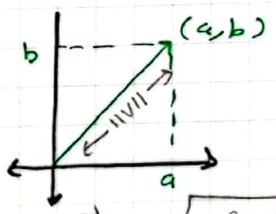
$\vec{v} = \langle v_1, v_2, v_3 \rangle$

$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

EX: $\vec{v} = \langle 3, 4, 2 \rangle$

$\|\vec{v}\| = \sqrt{9 + 16 + 4}$

$= \sqrt{29}$ (Mag)



$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$

(اصل القانون فيثاغورس)

Unit vector:

a vector with $\|\vec{U}\| = 1$

EX: Is $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \rangle$ unit.v

$\sqrt{\frac{1}{2} + \frac{1}{2} + 0} = 1$ ✓ unit vector

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Remark

$\|k\vec{v}\| = |k|\,\|\vec{v}\|$ ← proof

$\vec{V} = \langle V_1, V_2, V_3 \rangle$ L.H.S → left hand side

$\|\langle kV_1, kV_2, kV_3 \rangle\|$

$= \sqrt{(kV_1)^2 + (kV_2)^2 + (kV_3)^2} \Rightarrow \sqrt{k^2V_1^2 + k^2V_2^2 + k^2V_3^2}$

$\sqrt{k^2(V_1^2 + V_2^2 + V_3^2)}$

$\sqrt{k^2}\,\sqrt{(V_1^2 + V_2^2 + V_3^2)}$

$|k|\,\|\vec{V}\|$

END OF LECTURE

VECTORS

30/10/2023
Monday

- REMARK : Any vector in 3-space, can be written in form of $\hat{i}$, $\hat{j}$ and $\hat{k}$.

- EX: $\langle 3, 6, -2 \rangle = 3\hat{i} + 6\hat{j} - 2\hat{k}$.

Normalizing A vector

- To find a unit vector in same direction :

$$\vec{U} = \frac{\vec{V}}{\|\vec{V}\|}$$

- EX: Find a unit vector in same direction. To

$$\vec{v} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\|\vec{v}\| = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$\vec{U} = \frac{2\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{14}} = \frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{-1}{\sqrt{14}}k$$

- EX: Find :
  1. unit vector oppositly directed to $\vec{U}$
  2. vector with norm=10 with same direction.

$$\vec{V} = 6\hat{i} - 4\hat{j} + 2\hat{k}$$

① $\|\vec{V}\| = \sqrt{56}$

$$-\vec{U} = \frac{-6\hat{i} + 4\hat{j} - 2\hat{k}}{\sqrt{56}}.$$

② $\vec{U} = \left(\frac{6\hat{i} - 4\hat{j} + 2\hat{k}}{\sqrt{56}}\right) \times 10$

CALC III

30/10/2023

Monday

[DoT Product]
(scalar product)

DOT PRODUCT "Scalar Product"

⇒ $\vec{U}$, $\vec{V}$ Two vector $\vec{U}.\vec{V}$

⇒ LeT $\vec{U} = \langle U_1, U_2, 3 \rangle$, $\vec{V} = \langle V_1, V_2, V_3 \rangle$

Then $\vec{U}.\vec{V} = U_1 V_1 + U_2 V_2 + U_3 V_3$

EX: LeT $\vec{U} = \langle 3, 4, 6 \rangle$, $\vec{V} = \langle -1, 7, 5 \rangle$

$\vec{U}.\vec{V} = (3 \times -1) + (4 \times 7) + (6 \times 5) = 55$ #

Theorem: LeT $\vec{U}$, $\vec{V}$ be 2 vectors

① $\vec{U}.\vec{V} = \vec{V}.\vec{U}$

② $\vec{U}.(\vec{V} + \vec{W}) = \vec{U}.\vec{V} + \vec{U}.\vec{W}$

③ $\vec{V}.\vec{V} = \|\vec{V}\|^2 \Rightarrow \|\vec{V}\| = \sqrt{\vec{V}.\vec{V}}$

④ $\vec{U}.\vec{V} = \vec{0} = 0$ (scalar)

EX: Let $\vec{U} = \langle 1, -2, 2 \rangle$
$\vec{V} = \langle 2, 7, 6 \rangle$

Find the angle between $\vec{U}$ and $\vec{V}$.

$$\cos\theta = \frac{2 + -14 + 12}{\sqrt{9}\ \sqrt{88}} = 0$$

$\boxed{\theta = \frac{\pi}{2}}$ # App on Dot. P

Thus. $\vec{U}$ and $\vec{V}$ are normal prependiculer (orthogonal)

* If the dot product between 2 vectors = 0 then $\theta = \pi/2$ if neither of them = 0. *

Proof of theorem 3:
$\vec{V} = \langle V_1, V_2, V_3 \rangle$

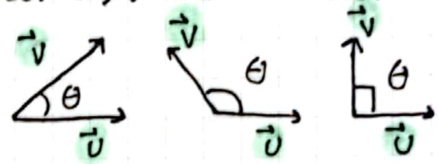
$$\vec{V}.\vec{V} = V_1.V_1 + V_2.V_2 + V_3.V_3$$
$$= V_1^2 + V_2^2 + V_3^2$$

$$(\|\vec{V}\|)^2 = \left(\sqrt{V_1^2 + V_2^2 + V_3^2}\right)^2$$

$$\|\vec{V}\|^2 = V_1^2 + V_2^2 + V_3^2 \ \#$$

Angle between 2 vectors.

Let $\vec{U}$, $\vec{V}$ be Two vectors.



$$\cos\theta = \frac{\vec{U}.\vec{V}}{\|\vec{U}\|.\|\vec{V}\|}, \quad 0 \leq \theta \leq \pi$$

$$\theta = \cos^{-1}\left(\frac{\vec{U}.\vec{V}}{\|\vec{U}\|.\|\vec{V}\|}\right)$$

$\cos\theta \Rightarrow$ isn't 1 To 1

يأخذ فترة لحل المشكلة

$0 \leq \theta \leq \pi$

EX: Find the angle between Two vectors.

Let $\vec{U} = 2\hat{i} - \hat{j} + 4\hat{k} = \sqrt{21}$
$\vec{V} = 4\hat{i} + 5\hat{k} = \sqrt{41}$

$$\cos\theta = \frac{8 + 20}{\sqrt{41}\ \sqrt{21}} = \frac{28}{\sqrt{41}\ \sqrt{21}}$$

$$\theta = \cos^{-1}\left(\frac{28}{\sqrt{41}\ \sqrt{21}}\right)$$

CALCULUS III
FOLLOWER !!

20/10/2023
Monday

Remark :

① If $\vec{U}\cdot\vec{V}=0$ then $\vec{U}=0$ or $\vec{V}=0$ or
$$\vec{U}\perp\vec{V}$$

② Show that the angle between $\vec{U}$ and $-\vec{U}=\pi$.

$\vec{U}=\langle U_1,U_2,U_3\rangle$; $-\vec{U}=\langle -U_1,-U_2,-U_3\rangle$

$$\cos\theta=\frac{\vec{U}\cdot-\vec{U}}{\|\vec{U}\|^2}=\frac{-U_1^2-U_2^2-U_3^2}{U_1^2+U_2^2+U_3^2}$$

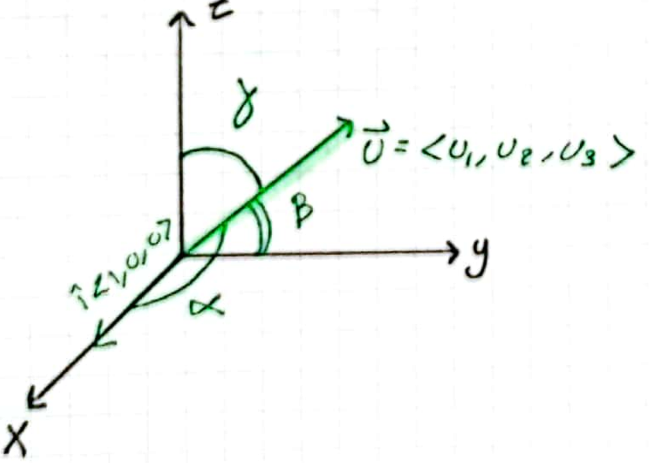
$$=\frac{-\cancel{(U_1^2+U_2^2+U_3^2)}}{\cancel{U_1^2+U_2^2+U_3^2}}=-1\ ,\ \theta=-\pi$$

EX: Show that if $\vec{U}+\vec{V}$ and $\vec{U}-\vec{V}$ are orthogonal, the $\vec{U}$ and $\vec{V}$ have the same length.

$$(\vec{U}+\vec{V})\cdot(\vec{U}-\vec{V})=0$$
$$(\vec{U}\cdot\vec{U})-\cancel{(\vec{U}\cdot\vec{V})}+\cancel{(\vec{V}\cdot\vec{U})}-(\vec{V}\cdot\vec{V})=0$$
$$(\|\vec{U}\|)^2-(\|\vec{V}\|)^2=0$$
$$(\|\vec{U}\|)^2=(\|\vec{V}\|)^2\Rightarrow\|\vec{U}\|=\|\vec{V}\|\ \#$$

Directional angles

1/11/2023
Wednesday



$$\cos\alpha=\frac{\vec{U}\cdot\hat{i}}{\|\vec{U}\|\cdot\|\hat{i}\|}\Rightarrow\frac{U_1}{\|\vec{U}\|}$$

$$\alpha=\cos^{-1}\left(\frac{U_1}{\|\vec{U}\|}\right)\qquad\beta=\cos^{-1}\left(\frac{U_2}{\|\vec{U}\|}\right)$$
$$\gamma=\cos^{-1}\left(\frac{U_3}{\|\vec{U}\|}\right)$$

CALCULUS III FOLLOWER

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wednesday

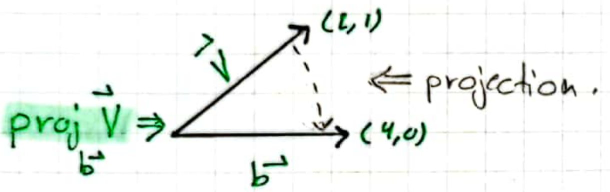
- EX: find the directional angles:
$V = 4\hat{i} + 7\hat{j} - 2\hat{k}$

$\|\vec{v}\| = \sqrt{69} \Rightarrow \cos\alpha = \frac{4}{\sqrt{69}}$

$\cos\beta = \frac{7}{\sqrt{69}}$

$\cos\gamma = \frac{-2}{\sqrt{69}}$

- projection of $\vec{v}$ on $\vec{b}$:



$$\vec{\gamma} = \left(\frac{\vec{V}.\vec{b}}{\|\vec{b}\|^2}\right)\vec{b}$$

- EX: Find the projection of $\vec{v}$ on $\vec{b}$ where:

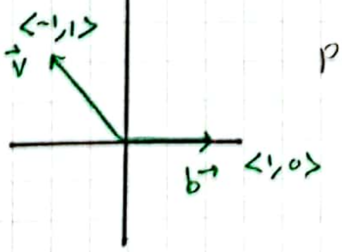
$\vec{V} = 4\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$

$$\text{proj}_{\vec{b}}\vec{V} = \left(\frac{\vec{V}.\vec{b}}{\|b\|^2}\right)\vec{b}$$

$$= \left(\frac{(12 + -2 - 1)}{11}\right) \times 3\hat{i} + \hat{j} - \hat{k}$$

$$= \frac{9}{11}(3\hat{i} + \hat{j} - \hat{k}).$$

- EX



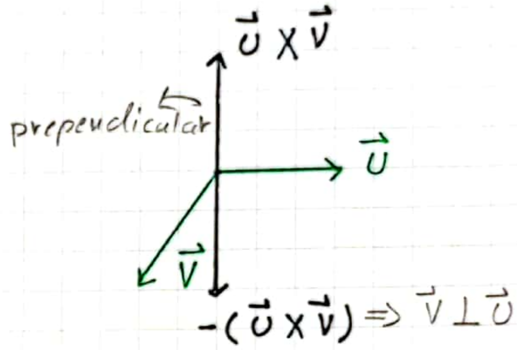
$$\text{proj}_{\vec{b}}\vec{V} = \left(\frac{-1}{1}\right)\langle 1,0\rangle$$
$$= \langle -1,0\rangle = -\hat{i}$$

11.4
Cross product

- Also know as: vector product.

- Let $\vec{U}$ and $\vec{V}$ be two vectors, then the cross product of $\vec{U}$ and $\vec{V}$, $\vec{U}\times\vec{V}$ is a vector perpendicular to $\vec{U}$ and $\vec{V}$

- No cross product in 2D.



Determinant: (det(A))
Matrix

$$\begin{vmatrix} 4 & 6 \\ 0 & 3 \end{vmatrix} = \begin{matrix} 4\times3 - 0\times6 \\ = 12 - 0 = 12\# \end{matrix}$$

$$\begin{array}{c} \begin{matrix} \oplus & \ominus & \oplus \end{matrix} \\ \begin{vmatrix} 0 & 1 & 2 \\ 3 & 1 & 5 \\ -2 & 4 & 6 \end{vmatrix} \end{array} = 0\begin{vmatrix} 1 & 5 \\ 4 & 6 \end{vmatrix} - 1\begin{vmatrix} 3 & 5 \\ -2 & 6 \end{vmatrix} + 2\begin{vmatrix} 3 & 1 \\ -2 & 4 \end{vmatrix}$$

$$= 0 - (18 + 10) + 2(12 + 2) = 0\#$$

- EX: Find $\vec{U}\times\vec{V}$:
$\vec{U} = \langle U_1, U_2, U_3\rangle$, $\vec{V} = \langle V_1, V_2, V_3\rangle$

$$\vec{U}\times\vec{V} = \begin{vmatrix} \hat{i} \ominus & \hat{j} \oplus & \hat{k} \\ U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \end{vmatrix}$$

$$= +\hat{i}\begin{vmatrix} U_2 & U_3 \\ V_2 & V_3 \end{vmatrix} - \hat{j}\begin{vmatrix} U_1 & U_3 \\ V_1 & V_3 \end{vmatrix} + \hat{k}\begin{vmatrix} U_1 & U_2 \\ V_1 & V_2 \end{vmatrix}$$

CALCULUS III

1/11/2023

wednesday

Cross product

EX: Find $\vec{U} \times \vec{V}$:

$$\vec{U} = 3\hat{i} - 2\hat{j} + \hat{k}$$
$$\vec{V} = 2\hat{i} + 5\hat{k}$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} - & \hat{j} + & \hat{k} \\ 3 & -2 & 1 \\ 2 & 0 & 5 \end{vmatrix}$$

$$\vec{U} \times \vec{V} = -10\hat{i} - 13\hat{j} + 4\hat{k} .$$

Also, $-(\vec{U} \times \vec{V})$ *
$10\hat{i} + 13\hat{j} - 4\hat{k}$ is perpendicular to $\vec{U}$ and $\vec{V}$.

* *

Remark:

1- $\vec{U} \cdot (\vec{U} \times \vec{V}) = 0$
$\vec{V} \cdot (\vec{U} \times \vec{V}) = 0$

2- $\hat{i} \times \hat{j} = \hat{k}$
$\hat{j} \times \hat{k} = \hat{i}$
$\hat{k} \times \hat{i} = \hat{j}$

3- $\hat{i} \times \hat{k} = -\hat{j}$
$\hat{j} \times \hat{i} = -\hat{k}$
$\hat{k} \times \hat{j} = -\hat{i}$

EX 2: Find the Area of the triangle with verticies:

$P_1(2,2,0)$
$P_2(-1,0,2)$
$P_3(0,4,3)$

$\overrightarrow{P_1P_2} = \langle -3, -2, 2 \rangle$
$P_1P_3 = \langle -2, 2, 3 \rangle$
$P_3P_2 = \langle -1, 4, 1 \rangle \leftarrow X$

$$\text{Area} = \frac{1}{2} \| \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} \|$$

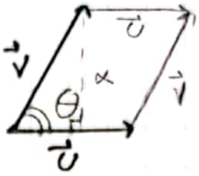
$$= \begin{vmatrix} \hat{i} - \hat{j} + \hat{k} \\ -3 \ -2 \ 2 \\ -2 \ 2 \ 3 \end{vmatrix} \Rightarrow -10\hat{i} + 13\hat{j} - 10\hat{k}$$

$$= \sqrt{100 + 169 + 100} = \frac{\sqrt{369}}{2}$$

Applications on cross.P

Monday 6/11

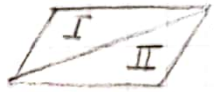
① Area of parallegram has $\vec{U}$ and $\vec{V}$ adjacent Sides



$A = \|\vec{U} \times \vec{V}\|$
$A = \|\vec{U}\| . \|\vec{V}\| \times \sin\theta$

$$\sin\theta = \frac{\alpha}{\|\vec{V}\|} \Rightarrow \alpha = \sin\theta \|\vec{V}\|$$

* Area of the triangle that $\vec{U}$ and $\vec{V}$ are adjacent sides



$$\Rightarrow A = \frac{1}{2} \|\vec{U} \times \vec{V}\|$$

$$A = \frac{1}{2} \|\vec{V}\| . \|\vec{U}\| \sin\theta$$

EX: Let $\vec{U} = 3\hat{i} + 2\hat{j} + \hat{k}$
$\vec{V} = 2\hat{i} - \hat{k}$.

$\Rightarrow$ Find the Area of parallegra

$$A = \|\vec{U} \times \vec{V}\|$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} - & \hat{j} + & \hat{k} \\ 3 & 2 & 1 \\ 2 & 0 & -1 \end{vmatrix}$$

$$= -2\hat{i} + 5\hat{j} - 4\hat{k}$$

$$\|\vec{U} \times \vec{V}\| = \sqrt{4 + 25 + 16}$$

$$= \sqrt{45} \ \#$$