CALCULUS III

13/12/2023

wednesday

Vector valued function

Rules

$\vec{r}(t)$. vector valued function
$f(t)$, Real valued function.

① $\frac{d}{dt}(\vec{r}_1(t) \pm \vec{r}_2(t)) = \frac{d}{dt}\vec{r}_1(t) + \frac{d}{dt}\vec{r}_2(t)$

② $\frac{d}{dt}(f(t).\vec{r}(t)) = f(t).\frac{d\vec{r}(t)}{dt} + \frac{df(t)}{dt}.\vec{r}(t)$

③ $\frac{d}{dt}(\vec{r}_1(t) \times \vec{r}_2(t)) =$

$\vec{r}_1(t) \times \frac{d\vec{r}_2(t)}{dt} + \frac{d\vec{r}_1(t)}{dt} \times \vec{r}_2(t)$

Exercises:

Let: $\vec{r}(t) = \cos(t)\hat{i} + \sin(t)\hat{j} + \hat{k}$
$f(t) = t^3$

① $\frac{d}{dt}(f(t).\vec{r}(t)) = f(t)\frac{d\vec{r}(t)}{dt} + r(t).\frac{df(t)}{dt}$

$= t^3(-\sin(t)\hat{i} + \cos(t)\hat{j}) +$
$3t^2(\cos(t)\hat{i} + \sin(t)\hat{j} + \hat{k})$.

سؤال فاينل

② $\frac{d}{dt}(\vec{r}(t) \times r'(t)) =$

$r' = -\sin(t)\hat{i} + \cos(t)\hat{j}$

$r \times r' = \begin{vmatrix} \hat{i} - & \hat{j} + & \hat{k} \\ \cos t & \sin t & 1 \\ -\sin t & \cos t & 0 \end{vmatrix}$

$= (-\cos(t))\hat{i} - (\sin(t)\hat{j}) + \hat{k}$
$= \sin(t)\hat{i} - \cos(t)\hat{j}$.

③ $\lim_{t\to 1}(\vec{r}(t).(\vec{r}'(t) \times \vec{r}''(t))) =$
(scalar triple product)

$r.(r' \times r'') = \begin{vmatrix} \hat{i} - & \hat{j} + & \hat{k} \\ \cos & \sin & 1 \\ -\sin & \cos & 0 \\ -\cos & -\sin & 0 \end{vmatrix}$

$= \sin^2(t) + \cos^2(t) = \boxed{1}$

④ $\lim_{t\to 0}(\vec{r}(t).\vec{r}'(t)) =$
$= -\sin(t)\cos(t) + \sin(t)\cos(t)$
$= 0$.

⑤ $\int \vec{r}(t)\, dt$
$= \int \cos(t)\hat{i} + \sin(t)\hat{j} + \hat{k}\, dt$
$= \sin(t)\hat{i} - \cos(t)\hat{j} + t\hat{k} + \vec{c}$

EX: $\int_0^1 \langle e^{2t}, e^{-t}, t \rangle\, dt$

$= \frac{e^{2t}}{2} - \frac{e^{-t}}{1} + \frac{t^2}{2} \Big]_0^1$

$= \frac{e^2}{2}\hat{i} - e^{-1}\hat{j} + \frac{t^2}{2}\hat{k} - \frac{1}{2}\hat{i} + \hat{j}$.

EX: Find the parametric equation of the line tangent to: $t = \frac{1}{3}$

$\vec{r}(t) = 2\cos(\pi t)\hat{i} + 2\sin(\pi t)\hat{j} + 3t\hat{k}$

① point : $\vec{r}(\frac{1}{3}) =$
$2\cos\frac{\pi}{3}\hat{i} + 2\sin\frac{\pi}{3}\hat{j} + \hat{k}$
$= \hat{i} + \sqrt{3}\hat{j} + k$
$pt = (1, \sqrt{3}, 1)$

② vector:
$r' = -2\pi\sin(\pi t)\hat{i} + 2\pi\cos(\pi t)\hat{j} + 3\hat{k}$

$r'(\frac{1}{3}) = -\pi\sqrt{3}\hat{i} + \pi\hat{j} + 3\hat{k}$

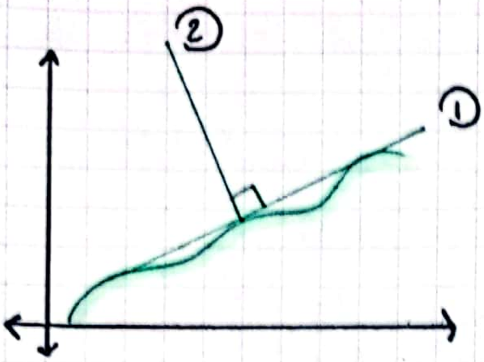
$x = 1 - \sqrt{3}\pi t$
$y = \sqrt{3} + \pi t$
$z = 1 + 3t$

CALCULUS III

[Tangent and normal $\vec{v}$]

13 / 12 / 2023

Wednesday



m = الميل

$\{ m_1 \times m_2 = -1 \}$

Let $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

Then:

① Unit tangent vector:

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|r'(t)\|}$$

② Unit normal vector:

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\|T'(t)\|}$$

EX: $\vec{r}(t) = \cos(t)\hat{i} + \sin(t)\hat{j} + t\hat{k}$

Find: 1 $\vec{T}(t) = \|r'\| = \sqrt{2}$

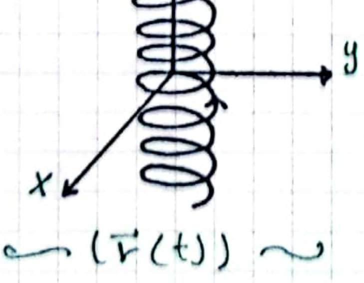
$r' = -\sin(t)\hat{i} + \cos(t)\hat{j} + \hat{k}$

$$\vec{T}(t) = \frac{-\sin(t)\hat{i}}{\sqrt{2}} + \frac{\cos(t)}{\sqrt{2}}\hat{j} + \frac{\hat{k}}{\sqrt{2}}.$$

2 $\vec{N}(t) = \|T'\| = 1/\sqrt{2}$

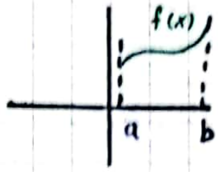
$$T' = \frac{-\cos(t)\hat{i}}{\sqrt{2}} + \frac{-\cos(t)\hat{j}}{\sqrt{2}}$$

$$\vec{N}(t) = -\cos(t)\hat{i} - \sin(t)\hat{j}.$$



~ ($\vec{r}(t)$) ~

Arc length: (Mon, 18)



$$L = \int_a^b \sqrt{1 + f'(x)^2}$$

IF $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{i} + z(t)\hat{k}$

Then arc length =

$$\Rightarrow \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\, dt$$

$$\Rightarrow \int_a^b \|r'(t)\|\, dt$$

Find arc length for:

$x(t) = \cos t$ from:
$y(t) = \sin t$ $t = 0$ to
$z(t) = t$ $t = \pi$.

$$L = \int \underbrace{\sqrt{\sin t^2 + \cos t^2 + 1^2}}_{\sin t^2 + \cos t^2 = 1}\, dt$$

$$L = \int_0^\pi \sqrt{2}\, dt$$

$$L = \sqrt{2}\, t \Big]_0^\pi \Rightarrow \sqrt{2}\,\pi$$

CALCULUS III
Chapter 13

18/12/2023
Monday

CHAP 13: Partial derivatives

13.1: functions of 2 or 3 variables.

$y = f(x)$

$y = x^2$	Domain $f(x)$
$y = \sin x$	Value of x interval
$y = \sqrt{x}$	Range of $f(x)$
$y = \ln x$	value of y.

functions of 2 Variables:

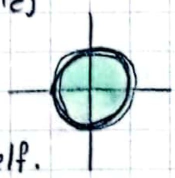
$* \; Z = f(x) \quad (\overset{x}{1}, \overset{y}{3}, \overset{z}{4})$
$f(x, y) = Z = x^2 + y$
$f(1, 3) = 1^2 + 3 = 4$

$* \; Z = f(x, y) = \sqrt{x + y}$
$f(1, 4) = \sqrt{5}$
$(1, 4, \sqrt{5})$

EX: Find the domain of:

1- $f(x, y) = \sqrt{1 - x^2 - y^2}$
$1 - x^2 - y^2 \geq 0$
$1 \geq x^2 + y^2$
$x^2 + y^2 = 1$ (circle)

Dom: inside the circle and the circle itself.
2 regions



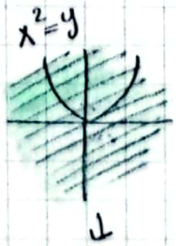
region: كل قيم x, y داخل الدائرة.

2- $f(x, y) = xy$

Dom = $R \times R$ or R^2
R of x — R of y
(xy plane كامل)

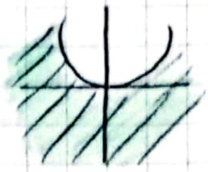
3- $f(x, y) = \dfrac{1}{x^2 - y}$

$x^2 - y = 0 \rightarrow x^2 = y$
Dom: $R^2 - \{x^2 = y\}$
→ xy plan كل

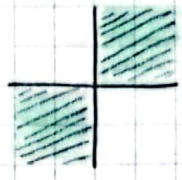


كل ال plan
إلا منحنى $y = x^2$.

4- $f(x, y) = \sqrt{x^2 - y}$
$x^2 - y = 0, \; x^2 = y$
Dom: $\mathbb{R}^2 = \{x^2 - y \geq 0\}$
or outside $x^2 = y$



5- $f(x, y) = \ln(xy)$
$xy > 0 \Rightarrow x, y \, (-)$
$x, y \, (+)$
(اما x, y موجبان او سالبان)



functions of 3 variables:

$w = f(x, y, z)$

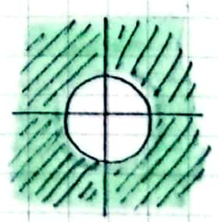
EX: let $w = f(x, y, z) = x^2 + 3y + 5z$
Find: $f(1, 2, 3)$:

$w = (1)^2 + 3 \times 2 + 5 \times 3 = 22$
$w = 22$
$\Rightarrow (\underset{x}{1}, \underset{y}{2}, \underset{z}{3}, \underset{w}{22})$

Find the domain of:

1. $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 9}$

$x^2 + y^2 + z^2 - 9 \geq 0$
$x^2 + y^2 + z^2 \geq 9$
Regon: كل إلي جوا الدائرة
مش بال Domain



Dom $= R^3 - \{x^2 + y^2 + z^2 < 9\}$

CALCULUS III FOLLOWER

18/12/2023

Mondays

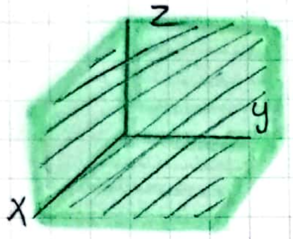
Find the domain :

2. $f(x,y,z) = 3x^2y^2 + z^3$

$Dom = \mathbb{R}^3$.

3. $f(x,y,z) = \ln(x)$

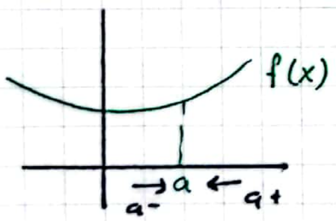
$x > 0$
x should be positive in all the values of x.
(positive x-axis).



limits and contin

→ In function of one variable.

$$\lim_{x \to a} f(x) \to \lim_{x \to a^+} f(x), \quad \lim_{x \to a^-} f(x)$$



* يمر من النقطة عدد لا نهائي المسارات (paths)

* Lim D.N.E IF 2 or more paths wasn't (=)
lim path (1) $\neq$ lim path (2)

Finding lim f(x) :

1- تعويض مباشر → ans = int (answer)
→ ans = 0/0
→ ans = int/0

$\boxed{1}$ IF ans = int
(يكون الرقم هو النهايات)

$\boxed{2}$ IF ans = int / 0
(D.N.E النهايات)
↓
غير موجودة

$\boxed{3}$ IF ans = 0/0
- نستخدم احد الطرق للإيجاد النهايات .
- تعويض / تحليل ----
← كالكولاس ١ و ٢ .
- طريقة جديدة باستخدام الـ paths .

Exercises :-

1. $z = f(x,y) = x^2y^2$

$\lim f(x,y) = \lim_{(x,y)\to(1,2)} x^2y^2$

$\lim (1)^2(2)^2 = 16$.

2. $\lim_{(x,y)\to(\frac{\pi}{4},1)} \sin(xy) =$

$\lim \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$.

3. $\lim \dfrac{x^4 - y^4}{x^2+y^2}, (x,y)\to(0,0)$

$\lim \dfrac{0-0}{0+0} = \dfrac{0}{0}$

$\lim \dfrac{(x^2-y^2)\cancel{(x^2+y^2)}}{\cancel{(x^2+y^2)}}$

$\lim 0 - 0 = 0$.

CALCULUS III

lim of 2 or 3 variables

20/12/2023

Wednesday

limit of 2 variable function:

1. $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{1-x^2+y^2} = \frac{0}{1} = 0$ Exist.

2. $\lim_{(x,y)\to(0,0)} \frac{x^4-16y^4}{x^2+4y^2} = \frac{0}{0}$

$\lim \frac{(x^2+4y^2)(x^2-4y^2)}{\cancel{(x^2+4y^2)}} = 0$ Exist.

3. $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \frac{0}{0}$

Path (1): x-axis $(y=0)$ $\lim_{y\to0} \frac{x(0)}{x^2+0} = 0$

Path (2): y-axis $(x=0)$ $\lim_{x\to0} \frac{0(y)}{0+y^2} = 0$

Path (3): $(y=x)$ $\lim_{x\to0} \frac{x^2}{2x^2} = \frac{1}{2}$

Path (1) $\neq$ Path (3) $\Rightarrow$ lim D.N.E

4. $\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2} = \frac{0}{0}$

Path (1): x-axis $(y=0)$ $\lim_{y\to0} = \frac{1}{2}$

Path (2): y-axis $(x=0)$ $\lim_{x\to0} = 1$

Path (1) $\neq$ Path (2) $\Rightarrow$ lim D.N.E

5. $\lim_{(x,y)\to(0,0)} \frac{\sqrt{x}\,y}{x+y^2} = \frac{0}{0}$

Path (1): x-axis $(y=0)$ $\lim_{y\to0} = 0$

Path (2): $y=\sqrt{x}$ $\lim_{x\to0} \frac{x}{2x} = \frac{1}{2}$

lim path (1) $\neq$ lim path (2)

lim D.N.E.

6. $\lim_{(x,y)\to(0,0)} \tan^{-1}\left(\frac{1}{x^2+y^2}\right) =$

$\lim \tan^{-1}\left(\frac{1}{0}\right) \Rightarrow \lim \tan^{-1}\infty$

$\lim = \frac{\pi}{2}$ Exist.

7. $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = 0$

تعويض :

$r^2 = x^2+y^2$

$r = 0+0 = 0$

$\lim_{r\to0} r^2\ln(r^2) = 2\lim_{r\to0} r^2\ln(r)$

لوبيتال :

$2\lim_{r\to0} \frac{\ln(r)}{\frac{1}{r^2}}$

$\cancel{2}\lim_{r\to0} \frac{-r^2}{\cancel{2}} = 0$ Exists.

8. $\lim_{(x,y)\to(0,0)} \frac{e^{-\frac{1}{\sqrt{x^2+y^2}}}}{\sqrt{x^2+y^2}} = \frac{0}{0}$

تعويض :

$r = \sqrt{x^2+y^2}$

$\lim_{r\to0} \frac{e^{-\frac{1}{r}}}{r}$

لوبيتال (1) :

$\lim_{r\to0} \frac{e^{-\frac{1}{r}}}{r^2}$

لوبيتال (2) :

$\lim_{r\to0} \frac{e^{-\frac{1}{r}}}{2r^3}$

$= \lim \frac{e^{-\frac{1}{0}}}{2(0)} = \frac{e^{-\infty}}{0} = 0$ Exists.

$\to (0)$

9. $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = \frac{0}{0}$

تعويض :

$r^2 = x^2+y^2$

$\lim_{r\to0} \frac{\sin(r^2)}{r^2} = 1$ Exists.

CALCULUS III

Continous function :)

20/12/2023

wednesday

REMINDER :

$\ln 1 = 0$, $\ln e = 1$

$\frac{9}{\infty} = 0$, $\frac{x}{\infty} = 0$, $e^{-\infty} = 0$

$\tan^{-1}(1) = \frac{\pi}{4}$, $\tan^{-1}(\infty) = \frac{\pi}{2}$

$\tan^{-1}(-\infty) = -\frac{\pi}{2}$, $\ln 0 = -\infty$

Continous function :

A function $f(x,y)$ is continous at (x_0, y_0) if :

1. $f(x_0, y_0)$ Exists
2. $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)$ Exists
3. $f(x_0, y_0) = \lim f(x,y)$

EX : IS $f(x,y)$ cont. ?

1. $f(x,y) = x^2 - 3y$ on $(1,2)$

A. $f(1,2) = -5$

B. $\lim\limits_{(x,y)\to(1,2)} f(x,y) = -5$

C. $f(1,2) = \lim f(x,y)$

Continous #.

2. $f(x,y) = \begin{cases} x^2 + 7y^2 & , \neq (0,0) \\ -4 & , = (0,0) \end{cases}$

is $f(x,y)$ cont. on $(0,0)$:

A. $f(0,0) = -4$

B. $\lim x^2 + 7y^2 = 0$

C. $-4 \neq 0$ Not cont. #

EX : IS $f(x,y,z)$ cont. at $(0,0,0)$?

$$f = \begin{cases} \frac{xyz}{x^2+y^4+z^4} & , \neq (0,0,0) \\ 10 & , = (0,0,0) \end{cases}$$

A. $f(0,0,0) = 10$

B. $\lim f(x,y,z)$

path 1 : x-axis , $y = z = 0$

$$= \lim_{x\to 0} \frac{0}{x^2} = 0$$

path 2 : Line equation:

$x = t^2$, $y = t$, $z = t$

$$= \lim_{t\to 0} = \frac{t^4}{3t^4} = \frac{1}{3}$$

lim path (1) $\neq$ lim path (2)

lim D.N.E , Not cont.

EX : $\lim\limits_{(0,0,0)} \frac{yz}{x^2+4y^2+4z^2}$

Path (1) : x-axis $(y = z = 0)$

$$\lim_{x\to 0} \frac{0}{x^2} = 0.$$

Path (2) : line equation:

$x = t$, $y = t$, $z = t$

$$\lim_{t\to 0} \frac{t^2}{14t^2} = \frac{1}{14}.$$

lim D.N.E path 1 $\neq$ path 2.

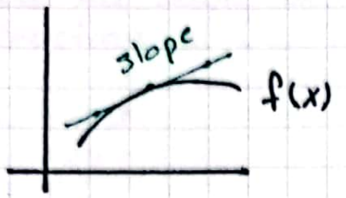
CALCULUS III

Partial Derivative

27/12/2023

Wednesday

SEC: 13.3 Partial derivatives



* Slope of tangent line : lim

* $\lim_{t \to 0} \frac{f(x+t) - f(x)}{t} = \frac{dx}{dt}$

$Z = F(X, y)$: $\frac{dz}{dx}$ = w.r.t x partial Derivative.

$\frac{dz}{dy}$ = w.r.t y

EX: find the following if
$f(x, y) = x^2 y + e^{xy}$

1. $f_x = \frac{df}{dx} = 2xy + e^x y$

2. $f_y = \frac{df}{dy} = x^2 + e^x$

3. $f_{xx} = \frac{d^2 f}{dx^2} = 2y + e^x y$

4. $f_{yy} = \frac{d^2 f}{dy^2} = 0$

5. $f_{xy} = \frac{d}{dy}\left(\frac{df}{dx}\right) = 2x + e^x$

6. $f_{yx} = \frac{d}{dx}\left(\frac{df}{dy}\right) = 2x + e^x$

7. $f_{xyx} = \frac{d}{dx}\left(\frac{d}{dy}\left(\frac{df}{dx}\right)\right)$

$= 2 + e^x$.

* $f_{xy} = f_{yx}$ * ← اذا الاقتران متصل .

EX: $f(x, y) = y^2 e^x + y$

Find: $\boxed{1}$ $f_{yyx} = \frac{d}{dx}\left(\frac{d}{dy}\left(\frac{df}{dy}\right)\right)$

$\Rightarrow 2ye^x + 1$
$\Rightarrow 2e^x + 1$
$\Rightarrow 2e^x$.

$\boxed{2}$ $f_{xyy} = \frac{d}{dy}\left(\frac{d}{dy}\left(\frac{df}{dx}\right)\right)$

$\Rightarrow y^2 e^x$
$\Rightarrow 2ye^x$
$\Rightarrow 2e^x$

EX: $\frac{d}{dP}\left(e^{\frac{-7P}{q}}\right) = \frac{-7}{q} * e^{\frac{-7P}{q}}$

EX: $\frac{d}{dq}\left(e^{\frac{-7P}{q}}\right) = \frac{7P}{q^2} * e^{\frac{-7P}{q}}$

EX: $f(x, y) = y^{-\frac{3}{2}} * \tan^{-1}\left(\frac{x}{y}\right)$

① $f_x = y^{-\frac{3}{2}} * \frac{1}{1 + \left(\frac{x}{y}\right)^2} * \frac{1}{y}$.

② $f_y = y^{-\frac{3}{2}} * \frac{1}{1 + \left(\frac{x}{y}\right)^2} * \frac{-x}{y^2}$

$+ \tan^{-1}\left(\frac{x}{y}\right) * \frac{-3}{2} y$.

Proof that $\tan^{-1}(x) = \frac{1}{1+x^2}$.

$\tan x = y$
$x = \tan y$
$(\sec^2 y)\, y' = 1$
$y' = \frac{1}{\sec^2 y} = \cos^2 y$
$y' = \frac{1}{1+x^2}$.

↑
مش مهم !!

CALCULUS III
FOLLOWER

- Let $f(x,y) = x^2y + y^3$:
Find the slope in x direction at $(1,-2)$:

$f_x = 2yx$
$f(1,-2) = -4$.

- EX: Show that:

$U(x,t) = \sin(x-ct)$
is a solution to

$$\frac{d^2U}{dt^2} = c^2 \frac{d^2U}{dx^2}$$

(wave equation)

* L.H.S.
$U(x,t) = \sin(x-ct)$

1. $\frac{dU(x,t)}{dt} = \cos(x-ct) * -c$

2. $\frac{d^2U}{dt^2} = c^2 * -\sin(x-ct)$
$\downarrow$
$\frac{d^2U}{dt^2} = c^2 * \frac{d^2U}{dx}$.

27/12/2023
Wednesday

Chain Rule

3/1/24 Wed

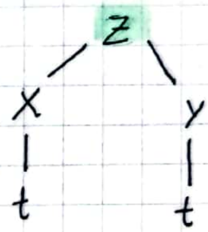
$y = f(x)$, $x = g(t)$

$$\frac{d(y)}{d(t)} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$\boxed{1}$ EX: $(\sin(x^2))' = 2x\cos x^2$

$\boxed{2}$ $z = f(x,y)$, $x = x(t)$
$y = y(t)$

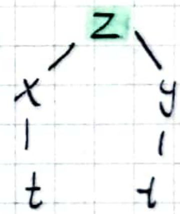
$\frac{dz}{dt}$?



$$= \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$$

* EX: $z = x^2y$, $x = t^2$, $y = t^3$

Find: $\frac{dz}{dt}$:



$$= \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$$

$= (2xy)(2t) + (x^2)(3t^2)$
$= 2t^2t^3 * 2t + t^4 * 3t^2$
$= 4t^6 + 3t^5$.

CALCULUS III

Chain Rule ü

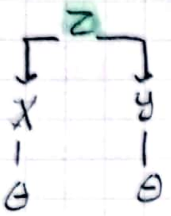
3/1/2024

wednesday.

EX : Find $dz/d\theta$

$Z = \sqrt{xy+y}$, $x = \cos\theta$

$\theta = 2$, $y = \cos\theta$



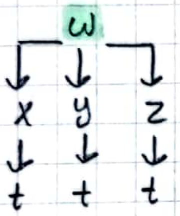
$$= \frac{dz}{dx}\cdot\frac{dx}{d\theta} + \frac{dz}{dy}\cdot\frac{dy}{d\theta}.$$

EX: Find dw/dt :

$w = x^2y + z^2xy + y$

$x(t) = t^2$, $y(t) = 5t$

$z = \sin t$

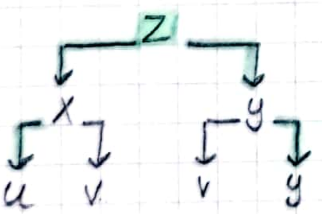


$$= \frac{dw}{dx}\cdot\frac{dx}{dt} + \frac{dw}{dy}\cdot\frac{dy}{dt} + \frac{dw}{dz}\cdot\frac{dz}{dt}.$$

EX: $Z = e^{xy}$, $x = 2u+v$

$y = u/v$:

Find: dz/du



$$= \frac{dz}{dx}\cdot\frac{dx}{du} + \frac{dz}{dy}\cdot\frac{dy}{du}$$

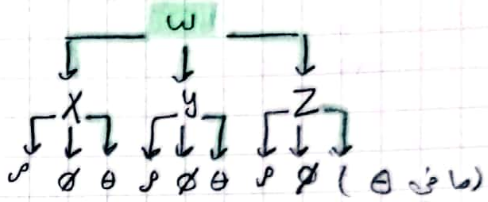
$$= ye^{xy}\cdot 2 + xe^{xy}\cdot\frac{1}{v}$$

$$= \frac{u}{v}e^{(2u+v)\cdot\frac{u}{v}} + \frac{(2u+v)e^{(2u+v)\cdot\frac{u}{v}}}{v}$$

EX: $w = x^2 + y^2 + z^2$, $x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$

Find: $dw/d\theta$:

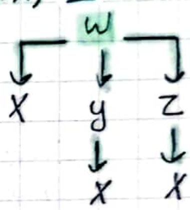


$$= w_x\cdot\frac{dx}{d\theta} + w_y\cdot\frac{dy}{d\theta}$$

$$= 2x(-\rho\sin\phi\sin\theta) + 2y(\rho\sin\phi\cos\theta)$$

EX : $w = xy + yz$, $y = \sin x$, $z = e^x$

Find : dw/dx

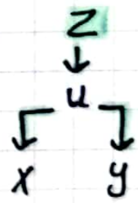


$$\Rightarrow \frac{dw}{dx} + \frac{dw}{dy}\cdot\frac{dy}{dx} + \frac{dw}{dz}\cdot\frac{dz}{dx}.$$

EX: Let $z = f(x^2 - y^2)$ show that:

$$y\frac{dz}{dx} + x\frac{dz}{dy} = 0$$

$z = f(u)$, $u = x^2 - y^2$



$$\frac{dz}{dx} = 2x\cdot\frac{dz}{du}$$

$$\frac{dz}{dy} = -2y\cdot\frac{dz}{du}$$

$$= y\left[2x\cdot\frac{dz}{du}\right] + x\left[-2y\cdot\frac{dz}{du}\right]$$

$$= 0 \#$$

CALCULUS III

Chain Rule ⍨

31 / 1 / 2024
wednesday

IF $Z = f(x, y)$ $\quad x = s + t$
$\qquad\qquad\qquad\quad y = s - t$

Show that:

$$\left(\frac{dz}{dx}\right)^2 - \left(\frac{dz}{dy}\right)^2 = \frac{dz}{ds} \cdot \frac{dz}{dt}$$

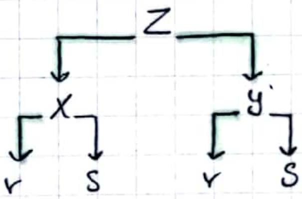
$$\frac{dz}{ds} = z_x \cdot \overset{\to 1}{\frac{dx}{ds}} + z_y \cdot \overset{\to 1}{\frac{dy}{ds}}$$

$$\frac{dz}{dt} = z_x \cdot \overset{\to 1}{\frac{dx}{dt}} + z_y \cdot \overset{\to -1}{\frac{dy}{dt}}$$

$$= (z_x + z_y)(z_x - z_y) = z_x^2 - z_y^2 \ \#$$

EX: $z = \sin(x + y)$, $x = r^2 + s^2$, $y = 2rs$

Find: $\frac{d^2z}{dr^2}$:



$$\frac{dz}{dr} = z_x \cdot \frac{dx}{dr} + z_y \cdot \frac{dy}{dr}$$

$$= z_x(2r) + z_y(2s)$$

$$\frac{d^2z}{dr^2} = 2z_x + (2r)\frac{d}{dr}\left(\frac{dz}{dx}\right) + (2s)\frac{d}{dr}\left(\frac{dz}{dy}\right).$$

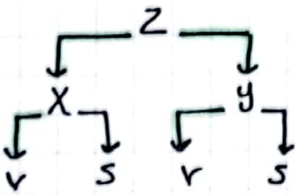
CALCULUS III

Directional Derivatives :)

8/1/2024
Monday

EX : IF $z = f(x, y)$, $x = r^2 + s^2$
$y = 2rs$

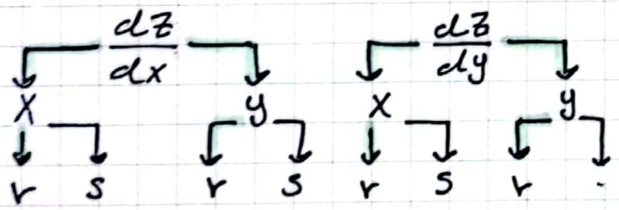
① $\dfrac{d^2 z}{dr^2}$:



$$\frac{dz}{dr} = \frac{dz}{dx} \cdot 2r + \frac{dz}{dy} \cdot 2s$$

$$\frac{d^2 z}{dr^2} = \frac{dz}{dx} \cdot 2 + 2r \cdot \frac{d}{dr}\left(\frac{dz}{dx}\right) + \frac{dz}{dy} \cdot 0 + 2s \frac{d}{dr}\left(\frac{dz}{dy}\right).$$

$$\frac{d^2 z}{dr^2} = 2\frac{dz}{dx} + 2r\left[\frac{d^2 z}{dr\,dx} \cdot \frac{dx}{dr} + \frac{d^2 z}{dr\,dx} \cdot \frac{dy}{dr}\right] + 2s\left[\frac{d^2 z}{dx\,dy} \cdot \frac{dx}{dr} + \frac{d^2 z}{dy\,dx} \cdot \frac{dy}{dr}\right].$$



Directional Derivatives and gradients :

* $D_{\vec{u}} f(x, y)$ at point (x_0, y_0)

$\vec{u}$ = unit vector $= \langle u_1, u_2 \rangle$
$f(x, y) = 2xy + y^2$
f_x = Derivative with respect to x

$$D_{\vec{u}} f(x, y) = f_x(x_0, y_0) \cdot u_1 + f_y(x_0, y_0) \cdot u_2$$

$$D_{\vec{u}} f(x, y, z) = f_x(x_0, y_0, z_0) u_1 + f_y(x_0, y_0, z_0) + f_z(x_0, y_0, z_0)$$
$$= \langle f_x, f_y, f_z \rangle \cdot \langle u_1, u_2, u_3 \rangle$$