CALCULUS III
Follower

Ex: Find $D_{\vec{u}} f(x,y,z)$:
$Pt = (1,-2,0)$
$f(x,y,z) = x^2 y - y z^3 + z$
$\vec{u} = \langle 2,1,-2 \rangle$

$$\vec{v} = \frac{\vec{u}}{\|\vec{u}\|} = \frac{\langle 2,1,-2\rangle}{3} = \langle \tfrac{2}{3}, \tfrac{1}{3}, \tfrac{-2}{3} \rangle$$

$f_x = 2xy \rightarrow f_x(1,-2,0) = -4$
$f_y = x^2 - z^3 \rightarrow 1$
$f_z = -3yz^2 + 1 \rightarrow 1$.

$$D_{\vec{u}} f(x,y,z) = \langle -4,1,1 \rangle \cdot \langle \tfrac{2}{3}, \tfrac{1}{3}, \tfrac{-2}{3} \rangle = -3 \ \#$$

Gradients ∇f

* Normal vector to the tangent plane of $f(x,y,z)$

$$\nabla f = \langle f_x(x_0,y_0,z_0), f_y(x_0,y_0,z_0), f_z(x_0,y_0,z_0) \rangle$$

* The tangent plane equation:

$$f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0) = 0.$$

Ex: Find the equation of the tangent plane to $x^2 + 4y^2 + z^2 = 18$ at $(1,2,1)$:

Normal vector to the tangent plane:

$f_x = 2x \rightarrow 2$, $f_y = 8y \rightarrow 16$, $f_z = 2z = 2$

$\nabla f = 2\hat{i} + 16\hat{j} + 2\hat{k}$

$= 2(x-1) + 16(y-2) + 2(z-1) = 0$.

→8
10/1/2024

Monday ;)

Critical points

① $f_x = 0$ and $f_y = 0$ or
f_x, f_y = D.N.E

② 2nd derivative test for max - min.

IF (a,b) C.pt : $f_x = 0$, $f_y = 0$

$$D = f_{xx}(a,b) \cdot f_{yy}(a,b) - f_{xy}^2(a,b)$$

① $D > 0$, $f_{xx} > 0$ min point
② $D > 0$, $f_{xx} < 0$ max point
③ $D < 0$ Saddle point

Find local min and max points and saddle pts :

$$f(x,y) = 4xy - x^4 - y^4$$

① $f_x = 4y - 4x^3 = 0 \Rightarrow y = x^3$
$f_y = 4x - 4y^3 = 0 \Rightarrow x = y^3$

② $y = (y^3)^3 \rightarrow y = y^9 \rightarrow y^9 - y = 0$

$y(y^8 - 1) = 0$, $\left.\begin{matrix}(0,0)\\(1,1)\\(-1,-1)\end{matrix}\right\}$ C.pts

Cpt	f_{xx}	f_{yy}	f_{xy}^2	D	point
(0,0)	0	0	16	$D<0$	Saddle
(1,1)	-12	-12	16	$D>0$	max
(-1,-1)	-12	-12	16	$D>0$	max

* f has a saddle pt at (0,0)
* f has a local max at (1,1)
* f has a local max at (-1,-1)

CALCULUS III

Chapter (14)

10/1/2024
wednesday

Multiple integrals :

Area : $\int_a^b f(x)\,dx$

Volum : $\pi\int_a^b (f(x))^2\,dx$

Arclength : $\int_a^b \sqrt{1+(f(x))^2}\,dx$

Sec 14.1 :

$\to f(x,y)$

$\therefore$ Volume under the surface $z=f(x,y)$ on A.

$$V=\iint_A f(x,y)\,dA$$

$dA = dx.dy$, or $dy.dx$

I. A : rectangular region.

Find V of $f(x,y)$ on the Region

$$R=\begin{cases} a \le x \le b \\ c \le y \le d \end{cases}$$

$$V=\iint_R f(x,y)\,dA$$

$$V=\int_{y=c}^{y=d}\int_{x=a}^{x=b} f(x,y)\,dx\cdot dy$$

$$V=\int_{x=a}^{x=b}\int_{y=c}^{y=d} f(x,y)\,dy\cdot dx$$

EX : Find the volume of $f(x,y)=x^2y$ on the region $\underset{(x)}{[0,1]} \times \underset{(y)}{[1,2]}$

$$V=\iint_R f(x,y)\,dA$$

$$V=\int_1^2\int_0^1 x^2y\,dx\,dy=\int_1^2 y\,\frac{x^3}{3}\Big]_0^1$$

$$V=\int_1^2 \frac{y}{3}\,dy=\frac{y^2}{6}\Big]_1^2$$

$$V=\frac{4}{6}-\frac{1}{6}=\boxed{\frac{3}{6}}\ \#$$

EX : Find : $\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\,dx\,dy$

$u=x^2+1 \Rightarrow \int_0^1\int_1^2 \frac{xy}{\sqrt{u+y^2}} * \frac{1}{2x}\,du\,dy$

$\Rightarrow \frac{1}{2}\int_1^2 \frac{y}{\sqrt{u+y^2}} \Rightarrow y\int_1^2 \frac{1}{\sqrt{u+y^2}}\,dy$

$\Rightarrow \frac{1}{2}\int_0^1 y\int_1^2 (u+y^2)^{\frac{1}{2}}\,du\,dy$

$\Rightarrow \frac{1}{2}\int_0^1 2y\sqrt{u+y^2}\Big]_1^2\,dy$

$\Rightarrow \int_0^1 y\left[(2+y^2)^{\frac{1}{2}}-(1+y^2)^{\frac{1}{2}}\right]dy$

$\Rightarrow \int_0^1 y\sqrt{2+y^2}\,dy-\int_0^1 y\sqrt{1+y^2}\,dy$

$w_1=2+y^2 \quad | \quad w_2=1+y^2$

$\Rightarrow \int_0^1 \frac{\sqrt{w_1}}{2}\,dw_1-\int_0^1 \frac{\sqrt{w_2}}{2}\,dw_2$

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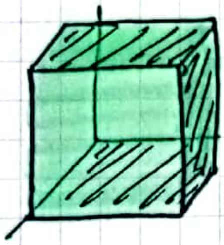
EX: $\int_0^5 \int_0^3 4 \, dx\, dy$



$\Rightarrow \int_0^5 12 \, dy$

$\Rightarrow 12y \Big]_0^5$

$\Rightarrow 60.$

EX: $\int_{\frac{\pi}{2}}^{\pi} \int_1^2 x\cos(xy) \, dy\, dx$

$\Rightarrow \int_{\frac{\pi}{2}}^{\pi} \frac{x \cdot \sin(xy)}{x} \, dx$

$\Rightarrow \int_{\frac{\pi}{2}}^{\pi} \sin(xy) \, dx \Big]_1^2$

$\Rightarrow \int_{\frac{\pi}{2}}^{\pi} \sin(2x) - \sin(x) \, dx \Big]$

$\Rightarrow \frac{-\cos(2x)}{2} + \cos(x) \Big]_{\frac{\pi}{2}}^{\pi}$.

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EX: $\displaystyle\int_0^1\int_{-x}^{x^2} y^2 x \, dy\,dx$

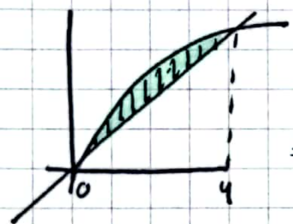
$\displaystyle\int_0^1 \frac{y^3}{3} x \Big|_{-x}^{x^2} dx$

$\displaystyle\frac{1}{3}\int_0^1 x[x^6 - - x^3]\, dx$

$\displaystyle\frac{1}{3}\int_0^1 x^7 + x^4 \, dx$

$\displaystyle\frac{x^8}{24} + \frac{x^5}{15}\Big|_0^1 .$

EX: $\displaystyle\iint_R (x+y)\, dA$ $\quad R = \left\{ \frac{x}{2} \leq y \leq \sqrt{x} \right\}$

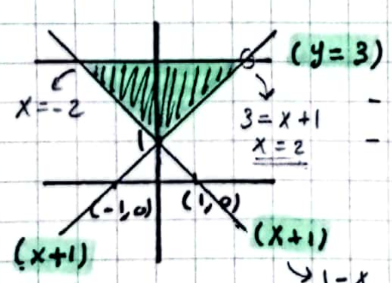


$\frac{x}{2} = \sqrt{x} \Rightarrow x = \{0, 4\}$

$\displaystyle = \int_0^4\int_{\frac{x}{2}}^{\sqrt{2}} (x+y)\, dy\, dx$

EX: $\displaystyle\iint_R (2x - y^2)\, dx\,dy$ $\quad R = y = 1-x$
$y = 1+x$
$y = 3$

(من المعادلات) $x = 0 \Rightarrow y = 1$



- y from 1 to 3
- x from 1-y to y-1

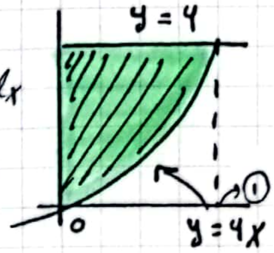
$\displaystyle = \int_1^3\int_{1-y}^{y-1} 2x - y^2 \, dx\,dy .$

or

$\displaystyle\int_{-2}^{0}\int_{1-x}^{3} 2x - y^2 \,dy\,dx + \int_0^2\int_{x+1}^{3} 2x - y^2 \, dy\,dx .$

EX: $\displaystyle\iint_R e^{-y^2}\, dy\, dx$

$\displaystyle\int_0^1\int_{4x}^{4} e^{-y^2} dy\,dx$



تكامل e^{-y^2} بالنسبة لـ dy
D.N.E

الحل : $\displaystyle\int_0^4\int_0^{y/4} e^{-y^2} dx\,dy$

$\displaystyle = \int_0^4 x e^{-y^2} \Big|_0^{\frac{y}{4}} dy$

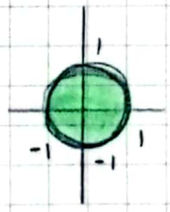
$\displaystyle = \int_0^4 y\, e^{-y^2} dy$, $w = y^2$.

EX: Find the volume of the solid bounded above: $z = 1 - y^2 - x^2$ and below by xy-plane, $z \geq 0$.

$\displaystyle = \iint 1 - x^2 - y^2 \, dA$

on the xy-plane $z = 0$,
$0 = 1 - y^2 - x^2 \;=\; x^2 + y^2 = 1$

from $x^2 + y^2 = 1$
$y = \pm\sqrt{1-x^2}$



$\displaystyle = \int_{-1}^{1}\int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} (1 - x^2 - y^2)\, dy\,dx$

EX: $\displaystyle\int_{\frac{1}{4}}^{1}\int_{x^2}^{x} \sqrt{\frac{x}{y}}\, dy\,dx$

$\displaystyle\int_{\frac{1}{4}}^{1}\int_{x^2}^{x} \sqrt{x}\,\sqrt{y^{-1}}\, dy\,dx$

$\displaystyle\int_{\frac{1}{4}} \sqrt{x} \cdot 2\sqrt{y}\Big|_{x^2}^{x} dx .$

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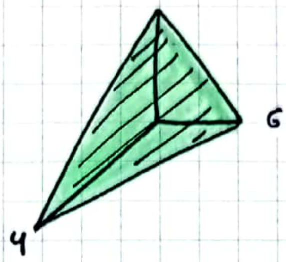
- Find the volume of $3x + 2y + 4z = 12$

$$z = \frac{12 - 3x - 2y}{4}$$

To Find x, $z = 0$ on the xy- plane:

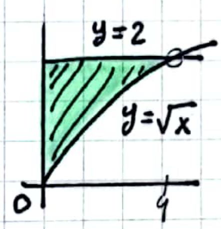


$$0 = 12 - 3x - 2y$$
$$0 = 3 - \frac{3}{4}x - \frac{2}{4}y$$
$$x = \frac{4}{3}\left(3 - \frac{1}{2}y\right)$$
$$\int_0^6 \int_0^{\left(\frac{4}{3}\left(3 - \frac{1}{2}y\right)\right)} \frac{12 - 3x - 2y}{4} \, dx\,dy .$$

- $\iint \sin(y^3)\, dA$ $\quad y = \sqrt{x}$, $y = 2$, $x = 0$

$2 = \sqrt{x}$, $x = 4$

$$\int_0^4 \int_{\sqrt{x}}^{2} \sin(y^3)\, dy\,dx$$



لا يمكن مكاملة $\sin(y^3)$ لـ dy لذلك :

$$\int_0^2 \int_0^{y^2} \sin(y^3)\, dx\,dy .$$

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[Double integral in polar coordinates]

* Double integral in polar coord.

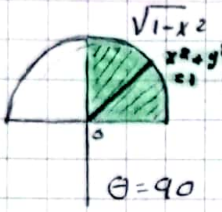
$$\iint_R f(x,y)\, dA$$

$$\Rightarrow \int_{\theta=}^{\theta=} \int_{r=f(\theta)}^{r=f(\theta)} f(r,\theta)\, r\, dr\, d\theta$$

$r^2 = x^2 + y^2$

$\tan\theta = \dfrac{y}{x}$

* EX: $V = 4\int_0^1 \int_0^{\sqrt{1-x^2}} 1-(x^2+y^2)\, dy\, dx$



$$4\int_0^{\frac{\pi}{2}} \int_0^1 (1-r^2)\, r\, dr\, d\theta$$

$\theta = 90$ $\quad y = \sqrt{1-x^2} \Rightarrow y^2 + x^2 = 1$

* EX: $\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} (x^2+y^2)^{\frac{3}{2}}\, dy\, dx$

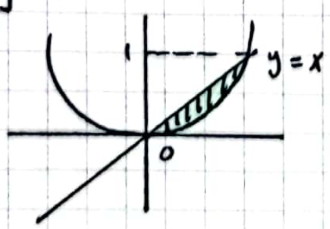
$$= \int_0^{\pi} \int_0^1 (r^2)^{\frac{3}{2}}\, r\, dr\, d\theta$$

حدود التكامل $\Rightarrow$

Final Q.

* EX: $\int_0^1 \int_y^{\sqrt{y}} \sqrt{x^2+y^2}\, dx\, dy$

$$= \int_0^{\frac{\pi}{4}} \int_0^{\tan\theta\sec\theta} \sqrt{r^2}\, r\, dr\, d\theta$$



$x^2 = y$
$x = y$

$x = r\cos\theta$
$y = r\sin\theta$

$r^2\cos^2\theta = r\sin\theta$

$r = \tan\theta\sec\theta$.

$\theta = \tan^{-1}(\frac{y}{x})$
$\tan^{-1}(1)$
$\theta = 45°$

14.5 Triple integrals

$$\iiint_G f(x,y,z)\, dv$$

$dv = dx\, dy\, dz$

EX: $\int_0^1 \int_1^2 \int_3^4 xyz\, dx\, dy\, dz$

$\Rightarrow \int_0^1 \int_1^2 \frac{x^2}{2} yz \Big|_3^4 dy\, dz$

$\Rightarrow \int_0^1 \int_1^2 yz\left[\frac{16}{2} - \frac{9}{2}\right] dy\, dz$

$\Rightarrow \int \frac{7}{2} \frac{y^2}{2} z \Big|_1^2 dz$

$\Rightarrow \int_0^1 z\, dz \left(\frac{7}{2}\left(\frac{4}{2} - \frac{1}{2}\right)\right.$.

EX: $\iiint z\, dv$

$$V = \begin{cases} 0 \le x \le y \\ 0 \le y \le 1 \\ 0 \le z \le \sqrt{1-y^2} \end{cases}$$

$$= \int_0^1 \int_{x=0}^{x=y} \int_0^{\sqrt{1-y^2}} z\, dz\, dx\, dy$$

$= \dfrac{1}{8}$ #

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* Volume of the solid:

$$\iiint_{C_a} 1 \, dv$$

EX: Find the volume of the solid inclosed by:

$z = 5x^2 + 5y^2$
$z = 6 - x^2 - y^2$ ← فيه z

$z = 0$
$5x^2 + 5y^2 = 6 - x^2 - y^2$ ← فيه x و y
$5x^2 + 6y^2 = 6$
$x^2 + y^2 = 1$ Circle

$$= \iiint 1 \, dz\,dx\,dy$$