CALCULUS III

1/11/2023

wednesday

Cross product

EX: Find $\vec{U} \times \vec{V}$:

$$\vec{U} = 3\hat{i} - 2\hat{j} + \hat{k}$$
$$\vec{V} = 2\hat{i} \qquad + 5\hat{k}$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} - & \hat{j} + & \hat{k} \\ 3 & -2 & 1 \\ 2 & 0 & 5 \end{vmatrix}$$

$$\vec{U} \times \vec{V} = -10\hat{i} - 13\hat{j} + 4\hat{k} .$$

Also, $-(\vec{U} \times \vec{V})$ *

$10\hat{i} + 13\hat{j} - 4\hat{k}$ is perpendicular to $\vec{U}$ and $\vec{V}$.

* *

Remark:

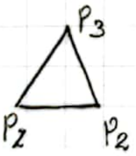
1- $\vec{U} \cdot (\vec{U} \times \vec{V}) = 0$
$\quad \vec{V} \cdot (\vec{U} \times \vec{V}) = 0$

2- $\hat{i} \times \hat{j} = \hat{k}$
$\quad \hat{j} \times \hat{k} = \hat{i}$
$\quad \hat{k} \times \hat{i} = \hat{j}$

3- $\hat{i} \times \hat{k} = -\hat{j}$
$\quad \hat{j} \times \hat{i} = -\hat{k}$
$\quad \hat{k} \times \hat{j} = -\hat{i}$

EX 2: Find the Area of the triangle with verticies:

$P_1(2, 2, 0)$
$P_2(-1, 0, 2)$
$P_3(0, 4, 3)$



$\overrightarrow{P_1P_2} = \langle -3, -2, 2 \rangle$
$P_1P_3 = \langle -2, 2, 3 \rangle$
$P_3P_2 = \langle -1, 4, 1 \rangle \leftarrow X$

$$Area = \frac{1}{2} \| \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} \|$$

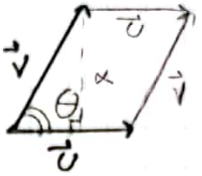
$$= \begin{vmatrix} \hat{i} - \hat{j} + \hat{k} \\ -3 \;\; -2 \;\; 2 \\ -2 \;\; 2 \;\; 3 \end{vmatrix} \Rightarrow -10\hat{i} + 13\hat{j} - 10\hat{k}$$

$$= \sqrt{100 + 169 + 100} = \frac{\sqrt{369}}{2}$$

Applications on cross.P

Monday 6/11

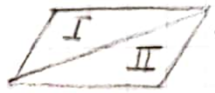
① Area of parallegram has $\vec{U}$ and $\vec{V}$ adjacent Sides



$$A = \| \vec{U} \times \vec{V} \|$$
$$A = \| \vec{U} \| \cdot \| \vec{V} \| \times \sin\theta$$

$$\sin\theta = \frac{\alpha}{\| \vec{V} \|} \Rightarrow \alpha = \sin\theta \| \vec{V} \|$$

* Area of the triangle that $\vec{U}$ and $\vec{V}$ are adjacent sides



$$\Rightarrow A = \frac{1}{2} \| \vec{U} \times \vec{V} \|$$

$$A = \frac{1}{2} \| \vec{V} \| \cdot \| \vec{U} \| \sin\theta$$

EX! Let $\vec{U} = 3\hat{i} + 2\hat{j} + \hat{k}$
$\vec{V} = 2\hat{i} - \hat{k}$.

$\Rightarrow$ Find the Area of parallegra[m]

$$A = \| \vec{U} \times \vec{V} \|$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} - & \hat{j} + & \hat{k} \\ 3 & 2 & 1 \\ 2 & 0 & -1 \end{vmatrix}$$

$$= -2\hat{i} + 5\hat{j} - 4\hat{k}$$

$$\| \vec{U} \times \vec{V} \| = \sqrt{4 + 25 + 16}$$

$$= \sqrt{45} \; \#$$

CIRCUITS 2

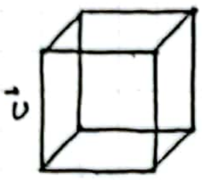
6/11/2023
Monday

Applications on cross prod

Section (5) 11.5

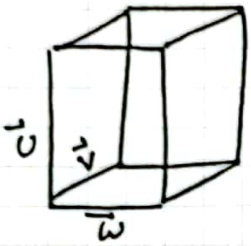
② Scalar triple product

let $\vec{U}, \vec{V}, \vec{W}$ be 3 vectors



Volume of the parallelpipal:

$$= |\vec{U}.(\vec{V}\times\vec{W})|$$



$$= \left|\begin{matrix} \oplus & \ominus & \oplus \\ U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \\ W_1 & W_2 & W_3 \end{matrix}\right|$$

■ EX: Find the scalar triple product (volume of the parallelpipal) where:

$$\vec{U} = 3\hat{i} - 2\hat{j} - 5\hat{k}$$
$$\vec{V} = \hat{i} + 4\hat{j} - 4\hat{k}$$
$$\vec{W} = 3\hat{j} + 2\hat{k}$$

$$V = |\vec{U}.(V\times W)| = \left|\begin{matrix} 3 & \ominus -2 & \oplus -5 \\ 1 & 4 & -4 \\ 0 & 3 & 2 \end{matrix}\right|$$

$$= 3(20) + (2\times 2) - (5\times 3)$$
$$= 49 \ \#.$$

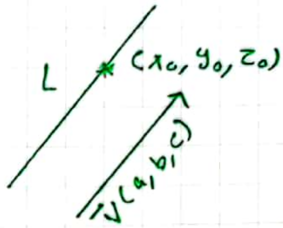
■ Remark:

$$\vec{U}.(\vec{V}\times\vec{W}) = \vec{V}.(\vec{W}\times\vec{U})$$
$$= \vec{W}.(\vec{U}\times\vec{V}).$$
$$= -\vec{V}.(\vec{U}\times\vec{W})$$

مادة الكويز

■ parametric equation of lines

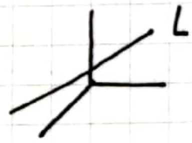
The para. equation of the line passing through (x, y, z) and paralleled to the vector $\langle a, b, c\rangle$ is:



$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + zt \\ -\infty < t < \infty \end{cases}$$

■ EX: Find the para. equation of the line passing $(1, 2, 3)$ and parallel to $\vec{V} = \langle 4, 5, 6\rangle$

$x = 1 + 4t$
$y = 2 + 5t$
$z = 3 + 6t$



■ EX: Find the para equation of the line passing through $(-2, 4, 5)$ and parallel to: $\vec{V} = \langle 2, 1, 10\rangle$

$x = x_0 + at \rightarrow -2 + 2t$
$y = y_0 + bt \rightarrow 4 + t$
$z = z_0 + zt \rightarrow 5 + 10t$

point on L, $t = 1$
$x = 0$
$y = 5$
$z = 15$

■ EX: Find the equation of the line passing through the pt's $(2, 4, -1)$ and $(5, 0, 7)$
P_1 P_2

$\overrightarrow{P_1P_2} = \langle 3, -4, 8\rangle$

$x = 2 + 3t$
$y = 4 + -4t$
$z = -1 + 8t$

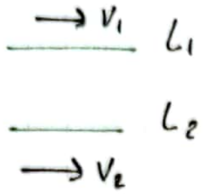
CALCULUS III

8/11/2023
wednesday

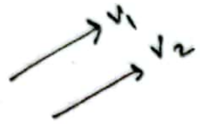
Relations between two Lines

1 2-space (2D)

1

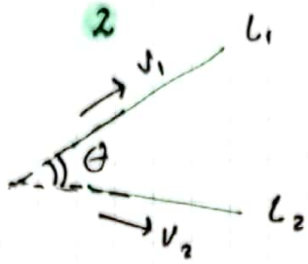


$L_1 // L_2$ if $v_1 // v_2$



$$\left\{ \vec{V}_1 = \lambda \vec{V}_2 \right\}$$

2



L_1 intersect L_2, v_1 intersect V_2, angle between two lines:

$$\left\{ \cos\theta = \frac{\vec{V}_1 \cdot \vec{V}_2}{\|V_1\| \|V_2\|} \right\}$$

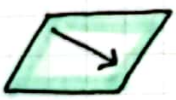
2 3-space (3D)

1 $L_1 // L_2 \Rightarrow V_1 // V_2$ $\{\vec{V}_1 = \lambda \vec{V}_2\}$

2 L_1 intersect $L_2 \Rightarrow \vec{V}_1$ intersect V_2, $\cos\theta = \frac{V_1 \cdot V_2}{\|V_1\| \cdot \|V_2\|}$.

3 Skew

(Diff Direction)



(Diff plane)

not parallel + Not intersect = skew

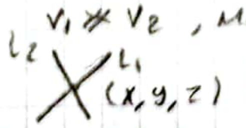
1 EX: Is L_1 $x = 3 + 4t$, $y = -1 + 5t$, $z = 9t$
L_2 $x = 5 + 8t$, $y = 11 + 10t$, $z = 1 + 18t$
parallel, intersection or skew

$V_1 = \langle 4, 5, 9 \rangle$ $V_2 \langle 8, 10, 18 \rangle$
$V_1 = \frac{1}{2} V_2 \Longrightarrow V_1 // V_2 \Rightarrow L_1 // L_2$ #

2 EX: L_1 $x = 4t$, $y = 1 - 2t$, $z = 2 + 2t$
L_2 $x = 1 + t$, $y = 1 - t$, $z = -1 + 4t$

① par, Intersect or skew

$v_1 = \langle 4, -2, 2 \rangle$ $v_2 = \langle 1, -1, 4 \rangle$
$v_1 \not\parallel v_2$, not parallel.



v_1 intersect v_2! #

$\Longrightarrow$

Intersection test:
$4t_1 = 1 + t_2 \Rightarrow 2t_1 + 1 = 2$
$1 - 2t_1 = 1 - t_2$ $\quad 2t_1 = 1$
$t_2 = 1$ $\quad t_1 = \frac{1}{2}$

$Zt_1 : 3$ $\quad Zt_2 = 3$
Intersection Pt = (2, 0, 3)

② The angle between L_1 + L_2:

$$\cos\theta = \frac{\vec{V}_1 \cdot \vec{V}_2}{\|\vec{V}_1\| \cdot \|\vec{V}_2\|}$$

3 EX: L_1 $x = 1 + 4t$, $y = 5 - 4t$, $z = -1 + 5t$

L_2 $x = 2 + 8t$, $y = 4 - 3t$, $z = 5 + t$

Solution: $\vec{V}_1 = \langle 4, -4, 5 \rangle$
$\vec{V}_2 = \langle 8, -3, 1 \rangle$

1. $\frac{4}{8} \neq \frac{-4}{-3} \neq \frac{5}{1} \Rightarrow$

$L_1 \not\parallel L_2$

2. X and Z
$1 + 4t_1 = 2 + 8t_2$
$(-1 + 5t_1 = 5 + t_2) \times -8$
$8 - 40t_1 = -40 - \cancel{8t_2}$
$1 + 4t_1 = 2 + \cancel{8t_2}$

$9 - 36t_1 = -38$.
$36t_1 = 47$ $\quad t_1 = \frac{47}{36}$

$1 + 4 \times \frac{47}{36} = 2 + 8t_2$
$1 + \frac{47}{9} - 2 = 8t_2$
$\frac{38}{72} = t_2$ $\quad yt_1 =$
$\quad yt_2 =$

L_1 not intersect L_2

L_1 skew L_2

CALCULUS III

11.6 PLANES

13/11/2023

Monday

Recall:

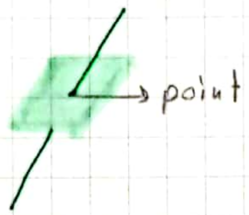
$x = x_0 + at$
$y = y_0 + bt$
$z = z_0 + ct$

Line equation ☺

$\langle x_0, y_0, z_0 \rangle$ point
$\langle a, b, c \rangle$ vector

EX: where does the Line intercect the plane?

$L: x = 2 - t$
$y = 3t$
$z = -1 + 2t$

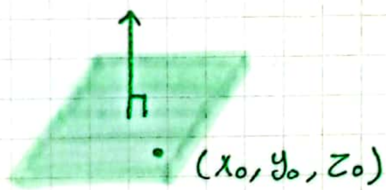


$2y + 3z = 6$

$2(3t) + 3(2t - 1) = 6$
$6t + 6t - 3 = 6 \quad t = \frac{9}{12} = \frac{3}{4}$

$x = 2 - \frac{3}{4}, \quad y = \frac{9}{4}, \quad z = -1 + \frac{3}{2}.$

11.6 planes:



To Find the equation of the plane we need a P (point) on this plane and a normal vector $\vec{n} = \langle a, b, c \rangle$

$\hookrightarrow a(x - x_0) + b(y - y_0) + c(z - z_0)$

$(x)', (y)', (z') \Leftarrow$ IMPORTANT

$\# \; a(x' - x_0) + b(y' - y_0) + c(z' - z_0) = 0$

EX 1: Find the equation of the plane passing through: $(2, -1, 4)$ and $\vec{n} = \langle 3, 5, 7 \rangle$

$S: a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
$3(x - 2) + 5(y + 1) + 7(z - 4) = 0$
$3(x) - 5y + 7z = 29.$

EX 2: For the plane Find
① $\vec{n}$
② Two points in the plane

plane: $5x - y + 7z + 5 = 0$

① $\vec{n} = \langle 5, -1, 7 \rangle$

② $x = 0, y = 0 \Rightarrow z = \frac{-5}{7}$
$\hookrightarrow (0, 0, \frac{-5}{7})$

$x = 0, z = 0 \Rightarrow y = 5$
$\hookrightarrow (0, 5, 0).$

EX 3: Find the equation of the plane passing through

$P_1(1, 2, 0)$
$P_2(0, 0, 3)$
$P_3(1, -1, 2)$

$\overrightarrow{P_1P_2} = \langle -1, -2, 3 \rangle$

$\overrightarrow{P_1P_3} = \langle 0, -3, 2 \rangle$

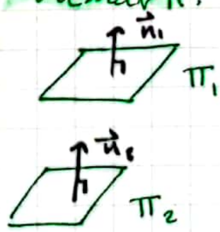
$\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$

$= \begin{vmatrix} \hat{i} - & \hat{j} + & \hat{k} \\ -1 & -2 & 3 \\ 0 & -3 & 2 \end{vmatrix}$

$= 9\hat{i} + \hat{j} - 5\hat{k}$

$\Rightarrow 9(x - 1) + (y - 2) + 3z = 0$

Remark:



$\Pi_1 \parallel \Pi_2$

if

$\vec{N}_1 \parallel \vec{N}_2$

$\vec{N}_1 = \lambda N_2$

CALCULUS III

[Follower]

13/11/2023

Monday

EX: consider the plane:

$\pi_1 = -9x - 6y + 3z = 4$

$\pi_2 = 3x + 2y - z = 20$

Is $\pi_1 \parallel \pi_2$?

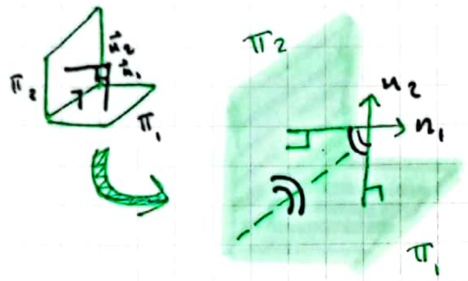
$\vec{n}_1 = \langle -9, -6, 3 \rangle$

$\vec{n}_2 = \langle 3, 2, -1 \rangle$

$\frac{\vec{n}_1}{\vec{n}_2} = \frac{-9}{3} = \frac{-6}{2} = \frac{3}{-1}$

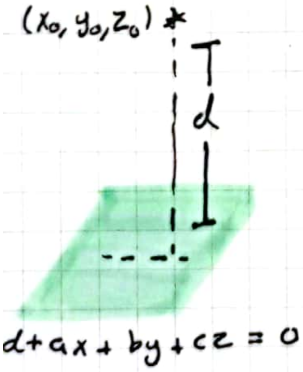
$\vec{n}_1 \parallel \vec{n}_2 \Rightarrow \pi_1 \parallel \pi_2$

Intersect:



$$\cos\theta = \frac{\vec{n}_1 \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|}$$

Distance between a point and a plane:



$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

* Find D between:
$(2, 1, 3)$ and
$3x - 3y + 5z + 7 = 0$

$$D = \frac{|6 - 3 + 15 + 7|}{\sqrt{9 + 9 + 5}}$$

$$D = \frac{25}{\sqrt{43}}$$

Relation between line and plane

WED 15/11

EX: Find the equation of the line of intersection for planes:

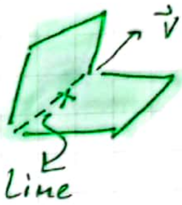
$3x + y - 5z = 0$

$x + y + z = 4$

Let $y = 0$

$3x - 5z = 0$

$x + z = -4$



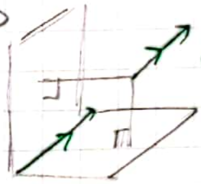
$x = -2.5$

$z = -1.5$

point: $(-2.5, 0, -1.5)$

$\vec{n}_1 = \langle 3, 1, -5 \rangle$

$\vec{n}_2 = \langle 1, 2, 1 \rangle$



$\vec{v} = \vec{n}_1 \times \vec{n}_2$

$= 11\hat{i} - 8\hat{j} + 5\hat{k}$

vector

$x = -2.5 + 11t$

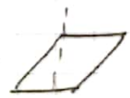
$y = -8t$

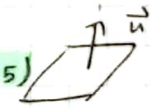
$z = -1.5 + 5t$

EX: Find the equation of the plane passing through:

$(1, 4, -5)$ and parallel to the plane $2x - 5y + 7z = 12$

$\vec{n}_1 = \vec{n}_2 = \langle 2, -5, 7 \rangle$





$2(x-1) + -5(y-4) + 7(z+5)$

$= 0$

$n = \langle 2, -5, 7 \rangle$

② Find the distance between π_1 and π_2

$$D = \frac{|\quad|}{\sqrt{\quad}} =$$

CALCULUS III

[Jallamer]

15/11/2023

WED

EX: Find the distance between two par planes:

$-2x + y + z = 0$
$6x - 3y - 3z = 5$

$(x, 0, 0) \Rightarrow -2x = 0$
$x = 0$ #

$\Pi_1 \Rightarrow P_1 = (0, 0, 0)$

D between P_1 and Π_2

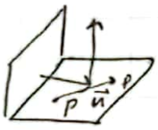
$D = \frac{| \quad |}{\sqrt{\quad}} =$

Final Q -

EX: Find the equation of the plane through the pt's and prependicular to the plane:

$P_1 (-2, 1, 4)$
$P_2 (1, 0, 3)$
$\Pi_1 (4x - y + 3z = 2)$

$\vec{n} = \langle 4, -1, 3 \rangle$
$\overrightarrow{P_1P_2} = \langle 3, -1, -2 \rangle$

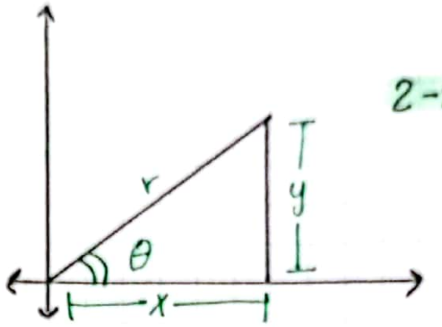


Cross product:

Polar coordinaties

MON 20/11

2-Space



From $(x, y) \rightarrow (r, \theta)$
Cartisian $\rightarrow$ polar

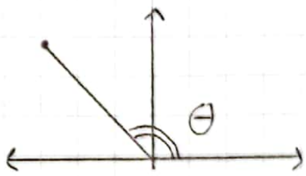
$\Rightarrow r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2 + y^2}$

$\Rightarrow \tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right)$

EX: Find the polar to the point: $(-2, 2\sqrt{3})$

S. $r = \sqrt{x^2 + y^2} \Rightarrow \sqrt{4 + 12} = 4$
$\theta = \tan^{-1}\left(\frac{y}{x}\right) \Rightarrow \tan^{-1}(-\sqrt{3})$
$\theta = \frac{2\pi}{3}$

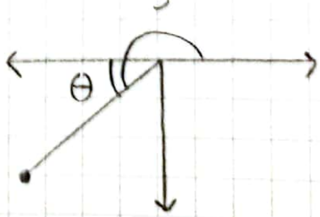
$\Rightarrow \left(4, \frac{2\pi}{3} \pm 2n\pi\right)$



EX: Find the polar cord for the point: $P(-2, -2\sqrt{3})$

S. $r = \sqrt{x^2 + y^2} = 4$
$\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$\Rightarrow \left(4, \frac{\pi}{3} \pm 2n\pi\right)$



CALCULUS III

20/4/2023

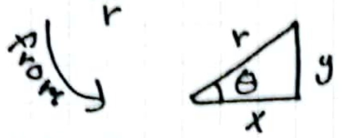
Monday

polar coordinates

From $(r, \theta) \longrightarrow (x, y)$

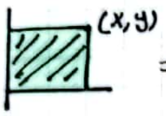
$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$



EX: Find rectangular for cord. $(6, \frac{\pi}{3})$

$x = 6\cos(\frac{\pi}{3}) \Rightarrow 3$
$y = 6\sin(\frac{\pi}{3}) \Rightarrow 3\sqrt{3}$

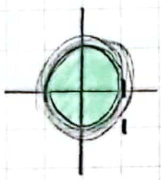


$= (3, 3\sqrt{3})$

EX: Sketch:

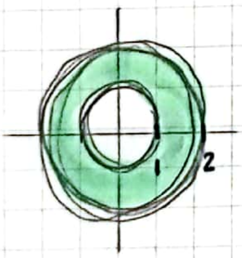
① $r = 1$

$r = \sqrt{x^2+y^2} \Rightarrow 1 = \sqrt{x^2+y^2}$
$1 = x^2+y^2 \Rightarrow$ unit circle



② A between $r=1$ and $r=2$

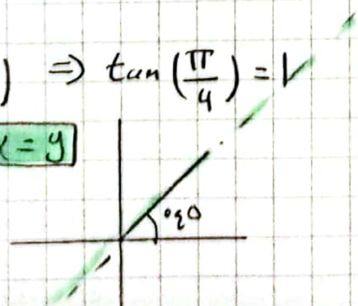
$r = 1 \Rightarrow 1 = x^2+y^2$
$r = 2 \Rightarrow 4 = x^2+y^2$



③ $\theta = \frac{\pi}{4}$

$\frac{\pi}{4} = \tan^{-1}(\frac{y}{x}) \Rightarrow \tan(\frac{\pi}{4}) = 1$

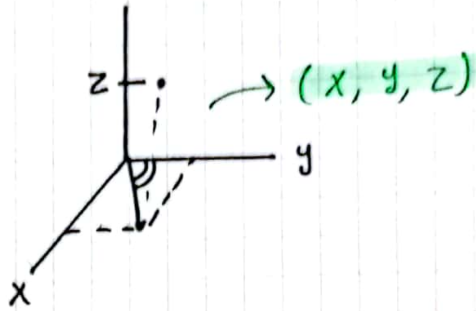
$1 = \frac{y}{x} \Rightarrow \boxed{x = y}$



polar coordinates in 3D: rectangular, cylindrical, spherical coordinates:

$r = \sqrt{x^2+y^2}$ $\quad \theta = \tan^{-1}(\frac{y}{x})$
$z = z$

From $(x, y) \longrightarrow (r, \theta)$



From $(r, \theta) \longrightarrow (x, y)$
$x = r\cos\theta$
$y = r\sin\theta$
$z = z$.

EX: Find the point in rectangular to cylindrical:
$P: (3, -3, -7)$

$r = \sqrt{9+9} = \sqrt{18}$
$\theta = \tan^{-1}(\frac{-3}{3}) = \tan^{-1}(-1)$
$\theta = \frac{7\pi}{4} \longrightarrow$ 4th Quarter

$\Rightarrow (\sqrt{18}, \frac{7\pi}{4} \pm 2n\pi, -7)$

EX: Find the point from cylin to rect:
$P: (2, \frac{\pi}{3}, 1)$

$x = 2\cos(\frac{2\pi}{3}) = -1$

$y = 2\sin(\frac{2\pi}{3}) = \sqrt{3}$

$z = 1$

$\Rightarrow (-1, \sqrt{3}, 1)$

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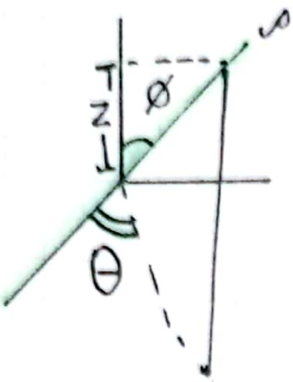
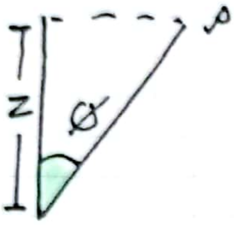
Spherical coordinates:



$\Rightarrow (\rho, \theta, \phi)$

$\Rightarrow$ proj of ρ to floor makes θ with $(+\hat{i})$.

$\rho = \sqrt{x^2+y^2+z^2}$
$\theta = \tan^{-1}(y/x)$
$\phi = \cos^{-1}(z/\rho)$



$\cos\phi = \frac{z}{\rho}$

$* \quad 0 \leq \theta \leq 2\pi$
$\quad 0 < \phi < \pi$

from $(\rho, \theta, \phi) \longrightarrow (x, y, z)$

$x = \rho \sin\phi \sin\theta$
$y = \rho \sin\phi \cos\theta$
$z = \rho \cos\phi$

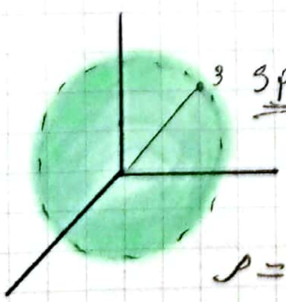
EX: write the pt $(2, \frac{\pi}{4}, \frac{\pi}{3})$ given in spherical to rect:

S. $x = \rho \sin\theta \sin\phi \Rightarrow 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$
$y = \rho \cos\theta \cos\phi \Rightarrow 2 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$
$z = \rho \cos\phi \Rightarrow 2 \times \frac{1}{2}$

$x = \frac{2\sqrt{3}}{2\sqrt{2}} \qquad y = \frac{\sqrt{3}}{2} \qquad z = 1$

[Midterm Question] $\Rightarrow \left(\frac{\sqrt{3}}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}, 1\right)$

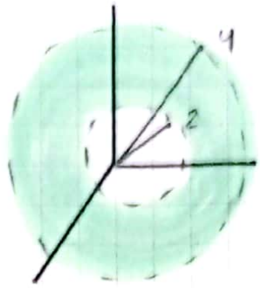
Sketch: $\rho = 3$



sphere

$\rho = x^2+y^2+z^2$

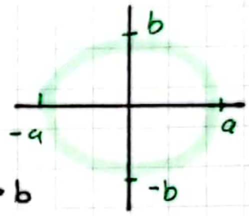
Sketch: $2 \leq \rho \leq 4$



$4 = x^2+y^2+z^2$
$16 = x^2+y^2+z^2$.

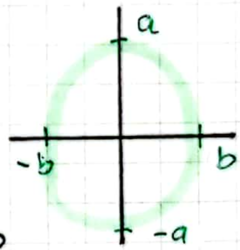
Conic Sections

1. Ellipse (قطع ناقص)



a>b

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ x-axis ellipse



a>b

$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ y-axis ellipse

$\Rightarrow 2x^2 + y^2 = 4$
$x^2 + \frac{y^2}{2} = 2 \Rightarrow \frac{x^2}{2} + \frac{y^2}{4} = 1$

$\Rightarrow$ y-axis ellipse #.

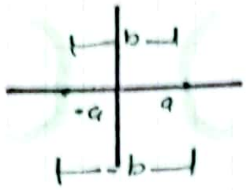
$a = 2 \qquad b = \sqrt{2}$.

Quadric Surfaces

22/11/2023
wednesday

2. Hyper pola (قطع زائد)

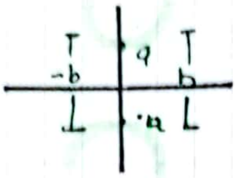
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



X-axis
Hyper pola

a is positive.

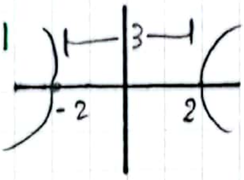
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



y -axis
Hyper pola

a is positive

EX : $\frac{x^2}{4} - \frac{y^2}{9} = 1$



3. Par bola :

$y = x^2$

$x = y^2$

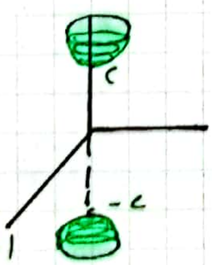
$x = -y^2$

$y = -x^2$

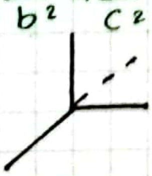
③ Hyperbulid of 2-sheets :

$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

X and y are Negative



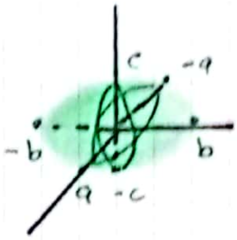
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



Quadric Surfaces

① $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Ellipsied

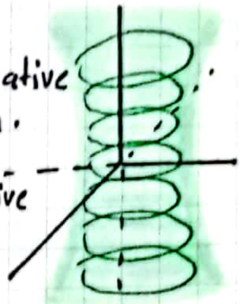


② Hyperbolid of 1-sheet :

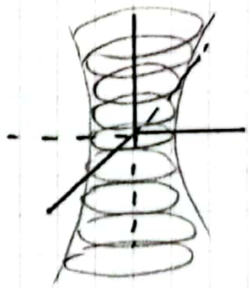
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

1 sheet
↳ z negative sighn.

z-negative



EX : $\frac{x^2}{4} + \frac{y^2}{2} - \frac{z^2}{5} = 1$



if x - negativ :

if y - neg

CALCULUS III

27/11/2023

Monday

Chapter (12)

Vector valued function

$\vec{F}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

EX: $\vec{F}(t) = 2t\hat{i} + (t)^2\hat{j} + 5\hat{k}$

$\vec{F}(2) = 4\hat{i} + 4\hat{j} + 5\hat{k}$.

Dom of $\vec{F}(t)$

① $f(x) = \sqrt{x} \geq 0$
$f(-1) = \sqrt{-1} \to X$

Dom $\in [0, \infty)$

② $f(x) = \dfrac{1}{(x-1)}$ $\neq 0$

Dom $\in (-\infty, \infty) - \{1\}$

③ $f(x) = \ln x$, $x > 0$

Dom $\in (0, \infty)$

$\text{Dom}(\vec{F}(t)) = \text{Dom}(x(t)) \cap \text{Dom}(y(t)) \cap \text{Dom}(z(t))$.

1 EX: Let $\vec{F}(t)$ be a function, Find the domain:

$\vec{F}(t) = \frac{1}{t}\hat{i} + \sqrt{t}\hat{j} + t^2\hat{k}$

$D(1/t) = R - \{0\}$
$D(\sqrt{t}) = [0, \infty)$
$D(t^2) = \mathbb{R}$

$-\infty$ 0 ∞

$D(\vec{F}) = (0, \infty)$.

2 $\vec{F}(t) = \ln\left(\frac{t-1}{t+1}\right)\hat{i} + \sqrt{1-t^2}\,\hat{j}$

$\text{Dom}(x(t)) \cap \text{Dom}(y(t))$

① $\ln\left(\frac{t-1}{t+1}\right)$ $\quad \frac{t-1}{t+1} = 0$

$t-1$ $\quad t+1$
$t=1$ $\quad t=-1$

$- - - - - +$ $t-1$
$- + + + + +$ $t+1$
$(+)$ -1 $(-)$ 1 $(+)$

$D = (-\infty, -1) \cup (1, \infty)$

② $\sqrt{1-t^2} \geq 0$
$1 - t^2 = 0$
$(1-t)(1+t) = 0$, $t = \mp 0$

$- -$ -1 $+ + + + +$ 1 $- -$

$D = [-1, 1]$

$D(\vec{F}) = \varnothing$ ⁐̈

3 $\vec{F}(t) = \sqrt[3]{\frac{t+1}{t^2-4}}\,\hat{i} + \log_7(x-7)\hat{j} + \frac{\hat{k}}{t-20}$

اصفار المقام $\quad \geq 0 \quad$ 20

limit of $\vec{F}(x)$

$\Rightarrow$ $F(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

$\hookrightarrow \lim_{t \to t_0} \vec{F}(t) = \lim_{t \to t_0} x + \lim_{t \to t_0} y + \lim_{t \to t_0} z$

EX: Let $F(t) = 2t\hat{i} + t^2\hat{j}$
Find $\lim_{t \to 3} F(t)$

$\lim_{t \to 3} F(t) = \lim_{t \to 3} 2t\hat{i} + \lim_{t \to 3} t^2\hat{j}$

$\lim_{t \to 3} F(t) = 6\hat{i} + 27\hat{j}$

CALCULUS III

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27/11/2023

Monday

2 Ex: $\vec{F}(t) = \left(\frac{t^2-1}{t-1}\right)\hat{i} + \frac{\sin(t-1)}{t^2-1}\hat{j} + 5\hat{k}$

Find $\lim_{t\to 1} F(t)$:

* $\lim \frac{t^2-1}{t-1} \Rightarrow \frac{\cancel{(t-1)}(t+1)}{\cancel{(t-1)}} = t+1 = 1$

* $\lim_{x\to 1} \frac{\sin(t-1)}{(t-1)(t+1)}$, $t-1 = x \Rightarrow \lim_{x\to 0} \frac{\sin x}{x(t+1)} = \lim_{x\to 1} \frac{1}{t+1} = \frac{1}{2}$

* $\lim F = \hat{i} + \frac{1}{2}\hat{j} + 5\hat{k}$

Finding first derivative

$\frac{dF(t)}{dt} = \overrightarrow{F'}(t) = x'(t)\hat{i} + y'(t)\hat{j} + z'(t)\hat{k}$

Ex: Find $\vec{F'}(t)$ and $\vec{F'}(\frac{\pi}{2})$:

$\vec{F}(t) = (2t^2+1)\hat{i} + \sin t\,\hat{j} + \frac{1}{t^3}k$

$F'(t) = 4t\,\hat{i} + \cos t\,\hat{j} + \frac{-3}{t^4}k$

$F(\frac{\pi}{2}) = 2\pi\,\hat{i} + 0 + -\frac{24}{\pi^3}\hat{k}$.

Ex: let $\vec{F}(t) = \sin t\,\hat{i} + \cos t\,\hat{j} + 5\hat{k}$

Find $\vec{F'}(t) \cdot \vec{F''}(t)$:

* $F'(t) = \cos t\,\hat{i} - \sin t\,\hat{j}$
* $F''(t) = -\sin t\,\hat{i} - \cos t\,\hat{j}$

$F' \cdot F'' = 0$ (orthogonal)