Subject: Calculus 2

Derivations

$\sec(x) \Rightarrow \frac{1}{\cos(x)}$

$\csc(x) \Rightarrow \frac{1}{\sin(x)}$

1. constant $\rightarrow$ zero
2. $ax \rightarrow a$
3. $x^n \rightarrow n x^{(n-1)}$
4. $ax^n \rightarrow a \cdot n(x^{n-1})$
5. $c(f(x)) \rightarrow c(\acute{f}(x))$
6. $f(x) \cdot g(x) \rightarrow (\acute{f}(x) \cdot g(x)) + (f(x) \cdot \acute{g}(x))$
7. $\frac{c}{g(x)} \rightarrow \frac{-c \cdot \acute{g}(x)}{(g(x))^2}$
8. $\frac{f(x)}{g(x)} \rightarrow \frac{(g(x) \cdot \acute{f}(x)) - (f(x) \cdot \acute{g}(x))}{(g(x)^2)}$

9. $\sqrt{f(x)} \rightarrow \frac{\acute{f}(x)}{2\sqrt{f(x)}}$
10. $\sin(x) \rightarrow \cos(x)$
11. $\cos(x) \rightarrow -\sin(x)$
12. $\tan(x) \rightarrow (\sec(x))^2$
13. $\cot(x) \rightarrow -(\csc(x))^2$
14. $\sec(x) \rightarrow \sec(x)\tan(x)$
15. $\csc(x) \rightarrow -\csc(x)\cot(x)$

16. $\sin^{-1}(x) \rightarrow \frac{1}{\sqrt{1-x^2}}$
17. $\tan^{-1}(x) \rightarrow \frac{1}{x^2+1}$
18. $\cos^{-1}(x) \rightarrow \frac{-1}{\sqrt{1-x^2}}$
19. $\cot^{-1}(x) \rightarrow \frac{-1}{x^2+1}$

20. $(g(x))^n \rightarrow n(g(x))^{n-1} \cdot \acute{g}(x)$
21. $f(g(x)) \rightarrow \acute{f}(g(x)) \cdot \acute{g}(x)$ $(f \circ g)$

Subject: Recall Tuesday / /

① $\int 1\,dx \Rightarrow x + c$

$\int e^{\pi^2}\,dy \Rightarrow e^{\pi^2} y + c$

$\int x\,dy \Rightarrow xy + c$

② $\int x^n\,dx \Rightarrow \frac{x^{n+1}}{n+1} + c$

$\int z^e\,dz \Rightarrow \frac{z^{e+1}}{e+1} + c$

$\int \sqrt[7]{y^5}\,dy \Rightarrow \int y^{\frac{5}{7}}\,dy \Rightarrow \frac{y^{\frac{5}{7}+1}}{\frac{5}{7}+1} + c$

$\int \frac{1}{y^{20}}\,dy \Rightarrow \int y^{-20}\,dy \Rightarrow \frac{y^{-19}}{-19}$

③ $\int x^{-1}\,dx \Rightarrow \int \frac{1}{x}\,dx \Rightarrow \ln|x| + c$

$\log_e$

مثال $\log_3 9 = 2$

④ $\int \frac{f'(x)}{f(x)}\,dx \Rightarrow \ln|f(x)| + c$

example $\int \frac{2x}{x^2+1}\,dx \Rightarrow \int \frac{2x}{x^2} + \frac{2x}{1} \Rightarrow \int \frac{2}{x} + 2x\,dx$ x غلط

$\Downarrow$

$\ln|x^2+1| + c$

$\int \frac{\cos t}{\sin t}\,dt \Rightarrow \ln|\sin t| + c$ $\int \cot$ نفسها

⑤ $\int e^x\,dx \Rightarrow e^x + c$ لأن مشتقتها نفسها

$\int e^{ax+b}\,dx \Rightarrow \frac{e^{ax+b}}{a} + c$

$\int a^x\,dx \Rightarrow \frac{a^x}{\ln a} + c$

$\int a^{bx+d}\,dx \Rightarrow \frac{a^{bx+d}}{b \ln a}$

Subject: / /

Examples:

e^{x^2} لا تُكامل

① $\int \pi^x \, dx \Rightarrow \frac{\pi^x}{\ln \pi} + C$

② $\int 7^{1+8x} \, dx \Rightarrow \frac{7^{1+8x}}{8 \ln|7|} + C$

⑥ $\int (ax+b)^n \, dx \Rightarrow \frac{(ax+b)^{n+1}}{(n+1)(a)}$ (لازم تكون قوة x تساوي 1)

Ex: $\int (2x+5)^{10} \, dx \Rightarrow \frac{(2x+5)^{11}}{11 \cdot 2} + C$

$\int \frac{1}{\sqrt[5]{(\pi x+7)^2}} \, dx \Rightarrow \int (\pi x+7)^{\frac{-2}{5}} \, dx$

$\frac{(\pi x+7)^{\frac{-2}{5}+1}}{(\frac{-2}{5}+1)\pi} + C$

x قوتها ①

⑦ $\int \sin(ax+b) \, dx \Rightarrow \frac{-\cos(ax+b)}{a} + C$

$\int \cos(ax+b) \, dx \Rightarrow \frac{\sin(ax+b)}{a} + C$

$\int \sec^2(ax+b) \, dx \Rightarrow \frac{\tan(ax+b)}{a} + C$

$\int \csc^2(ax+b) \, dx \Rightarrow -\frac{\cot(ax+b)}{a} + C$

$\int \sec(ax+b)\tan(ax+b) \, dx \Rightarrow \frac{\sec(ax+b)}{a} + C$

$\int \csc(ax+b)\cot(ax+b) \Rightarrow -\frac{\csc(ax+b)}{a}$

$\int \tan(ax+b) \, dx \Rightarrow \int \frac{\sin(ax+b)}{\cos(ax+b)} \Rightarrow \ln|\sec x| + C$

$\int \cot(ax+b) \, dx \Rightarrow \frac{\ln \sin(ax+b)}{a} + C$

Subject: _____ / /

Examples:

لا تُكامل e^{x^2}

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$\int \cot(ax+b) \, dx \Rightarrow \frac{\ln \sin(ax+b)}{a} + C$

Subject: / /

Example

① $\int \sec^2\left(\frac{4+6y}{10}\right) dy \Rightarrow \tan\left(\frac{4+6y}{10}\right) + C$

$\frac{6}{10}$

② $\int \sec x (\sec x + \tan x) dx \Rightarrow \int \sec^2(x)\, dx + \int \sec x \tan x\, dx$

$\tan(x) + \sec(x) + C$

Subject: Substitution rule / /

طريقة التعويض

$\int f'(x)\, g(f(x))\, dx \rightarrow u = f(x)$

$du = f'(x)\, dx \rightarrow dx = \frac{du}{f'(x)}$

* example:

① $\int e^x \cos(e^x)\, dx =$ اقتران ومشتقته بالسؤال

$u = e^x \Rightarrow \int \cancel{u'} \cos(u) \frac{du}{\cancel{e^x} \rightarrow u} \Rightarrow \int \cos(u)\, dy$

$du = e^x dx \Rightarrow dx = du$

$\sin(u) + C$ * لما يصير السؤال ما فيه x بكون الحل صحيح

$\sin e^x + C$ نرجع لأصلها

② $\int 2x\sqrt{1+x^2}\, dx \Rightarrow \int 2x\,(1+x^2)^{\frac{1}{2}}\, dx$ $u = 1+x^2$

$\int \cancel{2x}\, u^{\frac{1}{2}} \frac{du}{\cancel{2x}} = \int u^{\frac{1}{2}}\, du$ $du = 2x\,dx$

$\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$ $dx = \frac{du}{2x}$

$\boxed{\frac{(1+x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C}$

ما بصير ننقل الـ 2

③ $\int \frac{(\ln x)^2}{x}\, dx \rightarrow u = \ln x$ * مهم

$du = \frac{1}{x}\, dx$ $2\ln x = \ln x^2$

$\int \frac{u^2}{\cancel{x}}\, \cancel{x}\, du \Rightarrow \int u^2\, du$ $dx = x\,du$

$\frac{u^3}{3} + C \Rightarrow \frac{(\ln x)^3}{3} + C$

④ $\int \sec^2(x) \tan^5(x)\, dx$ $(\tan x)^5 = \tan^5 x$ *

$\int \cancel{\sec^2(x)}\, u^5 \frac{du}{\cancel{\sec^2(x)}} \Rightarrow \int u^5\, du$ $u = \tan x$

$\frac{u^6}{6}$ $du = \sec^2(x)\, dx$

$\frac{(\tan(x))^6}{6} + C$ $dx = \frac{du}{\sec^2(x)}$

⑤ $\int \frac{1}{\cos^2 t\sqrt{1+\tan^2 t}}\, dt = \int \frac{1}{\cos^2(t)\, \sec^2(t)}$ $\sec^2 = 1+\tan^2$ متطابقة

⇐ continue

Subject: / /

⑥ $\int \frac{x}{1+x^4}\,dx \Rightarrow \int \frac{x}{1+(x^2)^2}$ حتى أجعل x في البسط

$u = x^2$

$dy = 2x\,dx$

$dx = \frac{dy}{2x}$

$\int \frac{x}{1+u^2}\cdot\frac{du}{2x} \Rightarrow \frac{1}{2}\int \frac{1}{1+u^2}$ ← $\tan^{-1}$

$\frac{1}{2}(\tan^{-1}(x^2)) + c$

⑦ $\int \frac{x^2}{\sqrt{(1-x^6)}}\,dx$

$u = x^3$

$\int \frac{x^2}{\sqrt{1-(x^3)^2}}\,dx \Rightarrow \int \frac{x^2}{\sqrt{1-u^2}}\,\frac{du}{3x^2}$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

$\frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,du \Rightarrow \frac{1}{3}\cdot \sin^{-1}(x^3) + c$ ← $\sin^{-1}$

⑧ $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \csc^2(x)\, e^{\cot(x)}\,dx$

$u = \cot(x)$

$du = -\csc^2(x)\,dx$

$\int \csc^2(x)\, e^{u}\,\frac{du}{-\csc^2(x)} \rightarrow -\int e^u\,du$ $\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $dx = \frac{du}{-\csc^2(x)}$

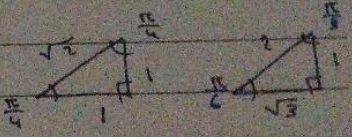
$-e^{\cot(x)}$

$= -\left(e^{\cot(\frac{\pi}{3})} - e^{\cot(\frac{\pi}{4})}\right)$

$-\left(e^{\frac{1}{\sqrt{3}}} - e^{1}\right)$

ملاحظة الأسس لا تتوقف جزئياً

مثلث أصول



⑨ $\int \frac{\sin(\frac{5}{x})}{x^2}\,dx = \int \frac{\sin u}{x^2}\cdot\frac{x^2\,du}{-5}$

$u = \frac{5}{x}$

$du = \frac{-5}{x^2}\,dx$

$dx = \frac{x^2\,du}{-5}$

⑩ $\int \sqrt{1+x^2}\cdot x^5\,dx \rightarrow \int \sqrt{u}\cdot x^3\,\frac{dy}{2x}$

$u = 1+x^2$

$dy = 2x\,dx$

$dx = \frac{du}{2x}$

$x^4 = (u-1)^2$

$\int \sqrt{u}\,(u^2 - 2u + 1)\,du$

$\int u^{\frac{1}{2}}(u^2 - 2u + 1)\,du$

$\int u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}$

Subject: Integration by parts Tuesday 24 / 10 / 2023

* Integration by parts :-

$\int x\, e^{x^2}\, dx$

sub → تعويض

$\int \ln x\, dx$

$\int x \sin x$

$\int x\, e^x\, dx$

* Parts : $\int u\, dv$ u: easy to differe

* $\int u\, dv = uv - \int v\, du$ dv: easy to integral

Example :

① $\int \underset{u}{x}\, \underset{dv}{e^x\, dx} \Rightarrow u = x \Rightarrow du = 1\,dx$

$\int dv = \int e^x\, dx \Rightarrow v = e^x$

$\int x\, e^x\, dx = x e^x - \int e^x dx \Rightarrow x e^x - e^x + C$

دائماً ناقص

② $\int (x+3)\cos(4x)\, dx$:

$u = x+3 \Rightarrow du = 1$

$dv = \cos(4x)dx \Rightarrow v = \frac{\sin(4x)}{4}$

$\int (x+3)\cos(4x)\, dx \Rightarrow \int (x+3)\left(\frac{\sin(4x)}{4}\right) - \int \frac{\sin(4x)}{4}\, dx$

$= (x+3)\left(\frac{\sin(4x)}{4}\right) + \frac{1}{4}\frac{\cos(4x)}{4} + C$

③ $\int x^2\, 2^x\, dx$

$u = x^2 \Rightarrow du = 2x\,dx$

$dv = 2^x\, dx \Rightarrow v = \frac{2^x}{\ln(2)}$

$\int x^2\, 2^x\, dx \Rightarrow \int \frac{x^2 \cdot 2^x}{\ln(2)} - \frac{2}{\ln(2)} \underbrace{\int x\, 2^x\, dx}_{\text{اجزاء كمان مرة}}$

$\int x^2\, 2^x dx = \frac{x^2\, 2^x}{\ln 2} - \frac{2}{\ln(2)}\left(\frac{x \cdot 2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\, dx\right)$

$= \frac{x^2\, 2^x}{\ln 2} - \frac{2}{\ln(2)}\left(\frac{x \cdot 2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}\right)$

$\int x\, 2^x\, dx$ تكامل

$x \longrightarrow 1dx$

$2^x\, dx \longrightarrow \frac{2^x}{\ln(x)}$

Subject: / /

example:

$$\int \underbrace{(x^3+4x)}_{u}\ \underbrace{\sin(6x)\ dx}_{dv}$$

U		dv
x^3+4x	$\oplus$	$\sin(6x)$
$3x^2+4$	$\ominus$	$\frac{-\cos(6x)}{6}$
$6x$	$\oplus$	$\frac{-\sin(6x)}{36}$
6	$\ominus$	$\frac{\cos(6x)}{36\cdot 6} \to 216$
0	$\oplus$	$\frac{\sin(6x)}{36\cdot 36}$
		$\frac{-\cos(6x)}{36\cdot 36\cdot 6}$

كثير حدود مع sin و cos

كثير حدود مع exponential

$$\left(x^3+4x\cdot\frac{-\cos(6x)}{6}\right)+\left(3x^2+4\cdot\frac{\sin(6x)}{36}\right)+6x\cdot\frac{\cos(6x)}{216}$$

$$-\left(\frac{6\sin(6x)}{1.296}\right)+C$$

example: ① $\int \underbrace{\ln x}_{u}\ \underbrace{dx}_{dv}$ $\qquad u=\ln x \to du=\frac{1}{x}dx$

$$\int \ln x\,dx = x\ln(x)-\int x\,\frac{1}{x}\,dx \qquad dv=1dx \to v=x$$

$$x\ln x - x + c$$

$\ln e = \cancel{zero}\ 1$

② $\int_e^{e^2} \ln x\,dx \Rightarrow x\ln x\Big|_e^{e^2} - \int_e^{e^2} x\,\frac{1}{x}\,dx$ $\qquad \ln 1 = zero$

$$(e^2\ln e^2)-(e\ln e) - x\Big|_e^{e^2}$$

$$e^2\ 2\cancel{\ln e}^{1} - (e\cancel{\ln e}^{1}) \qquad \downarrow (e^2-e)$$

$$2e^2 - e - e^2 + e$$

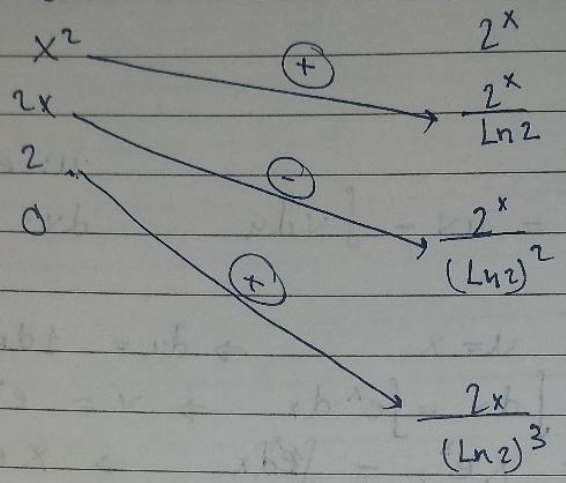
$$\boxed{e^2}$$

Subject: / /

طريقة أخرى اسرع

U ← بشتق حتى اوصل للصفر

dv ← بكامل نفس العدد



$$\int x^2\ 2^x\ dx \Rightarrow x^2\left(\frac{2^x}{\ln(2)}\right) - 2x\ \frac{2^x}{(\ln 2)^2} + 2\cdot\frac{2^x}{(\ln 2)^3} + C$$

$$\begin{array}{r} 3 \\ 36 \\ 6x \\ \hline 216 \end{array}$$