Subject: Power Series 16/ 1 /2024

$$\sum_{n=0}^{\infty} C_n x^n$$

$$= C_0 + C_1 x + C_2 x^2 + \cdots$$

1) $C_0 + C_1 + C_2 \rightsquigarrow$ coefficiant

2) x , variable متغير

Ex:

$$\sum_{n=0}^{\infty} x^n$$: geo ✓

$$1 + x + x^2 + x^3 + \cdots$$

If $x = \frac{1}{2} < 1$ con $\rightsquigarrow -1 < |x| < 1$ con

$x = 2 > 1$ div $\rightsquigarrow |x| > 1$ div ← | con | div →

$|x| < -1$ (-1 , 1)

Use root or ratio test for the interval of conv in power series:

Ex: For what values of x

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$$ con (اقتراض) $\Rightarrow < 1$

power series لأنها x فيها

لأنها x من كل القيم قوة n لأنها root test

Sol (ratio test): $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$

$$\lim_{n\to\infty} \left| \frac{(x-3)^{n+1}}{n+1} \cdot \frac{n}{(x-3)^n} \right| \rightarrow \lim_{n\to\infty} \left| \frac{(x-3)n}{n+1} \right| < 1 \Rightarrow (x-3)n \text{ تقارب}$$

$|x-3| < 1$ ← radius convergent

$-1 < x-3 < 1$: إذاً

$|a| < 1$

$2 < x < 4$ $\quad -1 < a < 1$

نعل test للقيمة (بالتقارب والتباعد)

نعوض قيم x

$x = 2$ → $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ → $\lim_{n\to\infty} \frac{1}{n} = 0$

$\frac{1}{n}$ decr $= \frac{-1}{n^2} < 0$

conv

$[2, 4)$

$x = 4$

$\sum_{n=1}^{\infty} \frac{(4-3)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ → $p = 1$ div

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

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Find interval of convergant and radius of convergant:

Ex: $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n (n+1)}$ → $\lim \left|\frac{a_{n+1}}{a_n}\right| < 1$ اختبار Ratio test

$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{3^{n+1}(n+2)} \cdot \frac{3^n (n+1)}{(-1)^n x^n} \right| < 1$ $(-1)^n$ تتقارب القيمة المطلقة

$\lim_{n\to\infty} \left| \frac{x(n+1)}{3(n+2)} \right| < 1 \rightarrow \left|\frac{x}{3}\right| < 1 \rightsquigarrow |x| < 3$ → radius

$-3 < x < 3$ الفترة المفتوحة

عوض بالأطراف $x = -3$

$\sum_{n=1}^{\infty} \frac{(-1)^n (-3)^n}{3^n (n+1)} = \frac{3^n}{3^n (n+1)} = \sum \frac{1}{n+1}$

div by L.C.T

$x = 3$

$\sum_{n=1}^{\infty} \frac{(-1)^n (3)^n}{3^n (n+1)} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)}$

$\lim = 0$ ← dec

conv by alternating test

$x \in (-3, 3]$

Ex: $\sum_{n=1}^{\infty} \frac{3^n x^n}{n!}$ → ratio $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$\lim_{n\to\infty} \left| \frac{3^{(n+1)} x^{(n+1)}}{(n+1)!} \cdot \frac{n!}{3^n \cdot x^n} \right| \rightarrow \lim_{n\to\infty} \left|\frac{3x}{n+1}\right| < 1 \rightarrow = 0 < 1$

interval of conv $(-\infty, \infty)$ $(R = \infty$ radius$)$

Ex: $\sum_{n=1}^{\infty} n^n (x-\pi)^n \rightarrow \sum_{n=1}^{\infty} \left|n^n (x-\pi)^n\right|^{\frac{1}{n}} < 1$ Root test

$\lim_{n\to\infty} \left| n(x-\pi) \right| < 1 \rightarrow (x-\pi) \lim_{n\to\infty} n < 1$

$\infty \not< 1$ div

$x = \pi \rightarrow$ conv

$x \neq \pi \rightarrow$ div

$R = 0$

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Taylor and Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad \text{(Maclaurin)}$$

أو اقترن نشر اكتبه على شكل كثير حدود
مشتقة وتم صفر بعض الاقتران الأصلي

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \quad \text{(Taylor)}$$

Example: find the Maclaurin series for $f(x) = e^x$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)\, x^n}{n!}$$

تعويض n

$$\frac{f^{(0)}(0)}{0!} x^0 + \frac{f^{(1)}(0)}{1!} x^1 + \frac{f^{(11)}(0)}{2!} x^2 + \frac{f^{(111)}(0)}{3!} x^3 + \dots$$

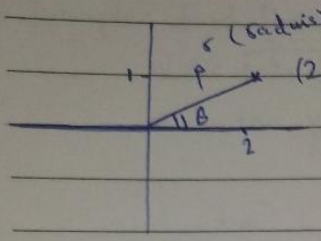
$$e^x = 1 + \frac{1}{1} x^1 + \frac{1}{2} x^2 + \frac{1}{3!} x^3 + \dots \leftarrow \text{كثير حدود}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^2 = \sum_{n=0}^{\infty} \frac{2^n}{n!} \rightarrow \text{conv}$$

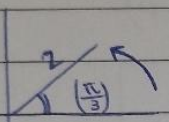
$$e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

Subject: Polar Coordinates — 21 / 1 / 2024



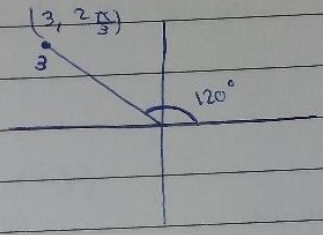
Ex: Sketch the point $(2, \frac{\pi}{3})$ in polar coordinates:

[1]



[2] Example:

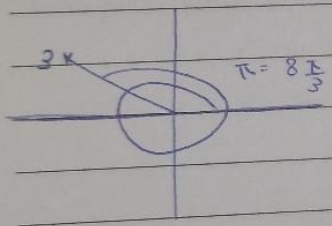
$(3, \frac{2\pi}{3})$



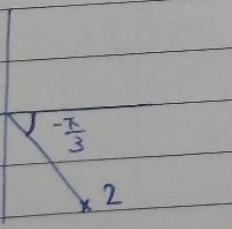
[3] $(3, \frac{8\pi}{3})$ → $\theta = 480$

$480 - 360 = 120\ (\frac{2\pi}{3})$

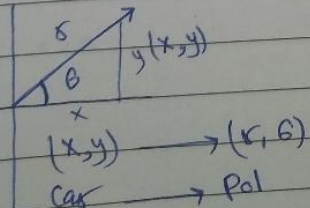
النقطة لها اتجاه لا نهائي



[4] $(2, -\frac{\pi}{3})$



تحويل



$(x,y) \rightarrow (r,\theta)$ Car → Pol

$r^2 = x^2 + y^2$ $\quad (x,y) \rightarrow (r,\theta)$

$r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x} \rightarrow \theta = \tan^{-1}\frac{y}{x}$

$(r,\theta) \rightarrow (x,y)$

$x = \cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$

$y = \sin\theta = \frac{y}{r} \rightarrow y = r\sin\theta$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

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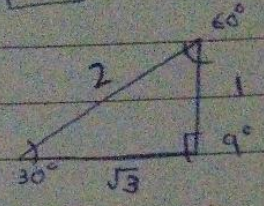
(r, θ)

Example: Find the Polar coordinate to the point $(\overset{x}{-2}, \overset{y}{2\sqrt{3}})$ مرة

1) $r = \sqrt{x^2 + y^2} \rightarrow \sqrt{(-2)^2 + (2\sqrt{3})^2} \rightarrow r = \sqrt{4+12} \rightarrow \boxed{r = 4}$

2) $\theta = \tan^{-1}\frac{y}{x} \rightarrow \tan^{-1}\frac{2\sqrt{3}}{-2} \rightarrow \tan^{-1} = -\sqrt{3}$

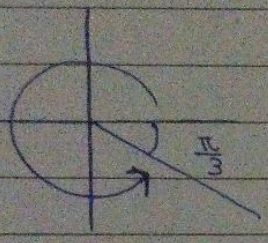


$\tan^{-1} = -\sqrt{3}$ ظل سالب X

زاوية مرجعية (دائماً مع محور x الموجب) $\frac{\pi}{3}$ 120

$(4, \frac{2\pi}{3} + 2n\pi)$ عدد لانهائي من الزاويات X tan سالب

نقاط السؤال:
الربع الرابع



360 - 60 = 300 $(5\frac{\pi}{3})$

$(4, \frac{5\pi}{3} + 2n\pi)$ عدد لانهائي من النقاط

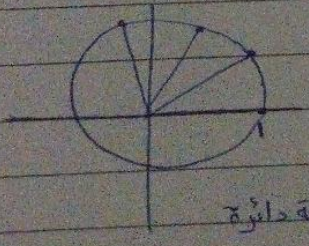
Example: Find rectangular (or cartizian) for the point $(\overset{r}{6}, \overset{\theta}{\frac{\pi}{3}})$

$x = r\cos\theta \rightarrow 6\cos\frac{\pi}{3} \rightarrow 6 \cdot \frac{1}{2} = 3$

$y = r\sin\theta \rightarrow 6\sin\frac{\pi}{3} \rightarrow 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$

$(3, 3\sqrt{3})$

Example: sketch $r = 1$ θ متغير



circle

Center (0,0)

$r = 1$; $r = \sqrt{x^2 + y^2}$

معادلة دائرة $1 = x^2 + y^2$ نصف القطر $= \sqrt{1^2} = 1$

$r = 3 \rightarrow 9 = x^2 + y^2$ نصف القطر $= \sqrt{9} = 3$