Subject: ____________ / /

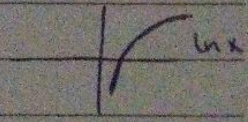
Is $\sum_{n=1}^{\infty} \frac{1}{n}$ conv or div ?

geo ☒

lim) $\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$ test failes → integral test

1. $a_n \geq 0 \rightarrow \frac{1}{n} \geq 0$ ✓
2. $a_n' = \frac{-1}{n^2} = - < 0$ ✓ decreasing
3. cont باثبتهن ① ✓ حققت الشروط ← اختبار التكامل

Improper $\int_1^{\infty} \frac{1}{x}\,dx \rightarrow \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx$



$\lim_{t\to\infty} \ln|x| \Big|_1^t = \ln|t| - \ln|1|$

$\ln\infty = \infty$ div

$\sum_{n=1}^{\infty} \frac{1}{n}$ div by integral test.

② $\sum_{n=1}^{\infty} \frac{1}{n^2}$ con or div ?

$\lim_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty} = 0$ test fail

✓ ① $a_n \geq 0$

✓ ② $a_n' = \frac{-2n}{n^4} < 0$

✓ ③ cont

$\int_1^{\infty} \frac{1}{n^2}\,dn \rightarrow \lim_{t\to\infty} \int_1^t \frac{1}{n^2}\,dn$

$\lim_{t\to\infty} \Big|_1^t \frac{-1}{n} = \lim_{t\to\infty} \frac{-1}{\infty} - \frac{-1}{1} = 0 + 1 = 1$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ conv series by integral test.

* $\sum_{n=1}^{\infty} \frac{1}{n^p}$: series $p > 1 \rightsquigarrow$ conv

$p \leq 1 \rightsquigarrow$ div

↓

$\int_1^{\infty} \frac{1}{x^p}\,dx$

* $\sum_{n=1}^{\infty} ar^n$ geo ☒

$|r| < 1 \rightarrow$ conv

* $\sum_{n=1}^{\infty} \frac{1}{n^p} \rightarrow$ p seris

$p > 1 \rightsquigarrow$ conv

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series مهمة

Ex: Is $\sum_{K=1}^{\infty} \frac{4}{6K^3}$ con or div ?

$p=3>1 \rightsquigarrow$ conv الثوابت مابتأثر

conv by integral test

Ex: $\sum_{K=1}^{\infty} Ke^{-K^2} \rightsquigarrow \sum_{K=1}^{\infty} \frac{K}{e^{K^2}}$ geo $\boxtimes$

p. series

$\lim_{K\to\infty} \frac{K}{e^{K^2}} = \frac{\infty}{\infty} \rightarrow$ لوبيتال $= \frac{1}{2K e^{K^2}} = \frac{1}{\infty} = 0$

test faile

* by integral test $\rightarrow \int_1^{\infty} Ke^{-K^2} \rightarrow \int_1^{\infty} xe^{-x^2} dx$

$\lim_{t\to\infty} \int_1^t x e^{-x^2} dx \rightarrow \lim \int_1^t x\, e^{-u} \frac{du}{2x} \rightarrow \frac{1}{2} \lim_{t\to\infty} -e^{-u} \Big|_1^t$

$u = x^2$

$du = 2x\, dx$

$dx = \frac{du}{2x}$

$-\frac{1}{2} \lim_{t\to\infty} [e^{-t} - e^{-1}] \rightarrow -\frac{1}{2} \lim_{t\to\infty} [\frac{1}{e^t} - \frac{1}{e}]$

$= -\frac{1}{2}(-\frac{1}{e}) = \frac{1}{2e} \rightarrow$ conv be integral test

Ex: $\sum_{K=5}^{\infty} 7K^{-1,001} \rightarrow \sum_{K=5}^{\infty} \frac{7}{K^{1001}}$

P. series

$P = 1,001 > 1 \rightsquigarrow$ conv

الgeome تكون conv اذاكانت اقل من 1

$\sum_{K=1}^{\infty} \frac{\tan^{-1} K}{K^2+1}$

Recall:

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ex: Is conv or div ?

$\sum_{K=1}^{\infty} \left(\frac{2}{3}\right)^{K-1} + \frac{1}{K} \rightarrow \sum_{K=1}^{\infty} \left(\frac{2}{3}\right)^{K-1} + \sum_{K=1}^{\infty} \frac{1}{K}$ geo

$\left(\frac{2}{3}\right)^{K} \cdot \left(\frac{2}{3}\right)^{-1}$ $\quad$ $K \leq 1 \rightarrow$ div

$r = \frac{2}{3} < 1 \rightarrow$ conv

conv + div $\rightsquigarrow$ div

ex: $\sum_{n=1}^{\infty} 5 \rightsquigarrow$ div

* limit compar sim test (L.C.T)

let a_n, $\sum b_n$, $a_n, b_n \geq 0$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \square$$

if $\sum b_n$ con $\rightsquigarrow \sum a_n$ conv

" " " div $\rightsquigarrow$ " " div

ex: $\sum_{n=1}^{\infty} \frac{n^2+1}{n^4+5} \longrightarrow \lim_{n\to\infty} \frac{n^2+1}{n^4+5}$ = zero (test failes)

ex: $\sum_{n=1}^{\infty} b_n = \frac{n^2}{n^4} \rightsquigarrow \sum_{n=1} \frac{1}{n^2}$ $\quad$ $p=2 \rightsquigarrow p>1$

conv

$$\lim_{n\to\infty} \frac{a_n}{b_n} \rightsquigarrow \lim_{n\to\infty} \frac{\frac{n^2+1}{n^4+5}}{\frac{1}{n^2}} = \frac{n^4+n^2}{n^4+5} = 1$$

conv

Recall:

$\sum a_n, \sum b_n$ series

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim > 0$

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Ex: $\sum_{K=1}^{\infty} \frac{1}{\sqrt{K}+1}$ لما div ← خذ lim اللي جوا

1) div test: $\lim_{K\to\infty} \frac{1}{\sqrt{K}+1} = \frac{1}{\infty} = 0$ test failes.

2) LCT: $\sum b_K = \sum_{K=1}^{\infty} \frac{1}{\sqrt{K}} \rightsquigarrow$ div لاتوزع المقام على البسط

Pseries $P = \frac{1}{2} < 1$

$\frac{a_n}{b_n}$ $\lim_{K\to\infty} \frac{\frac{1}{\sqrt{K}+1}}{\frac{1}{\sqrt{K}}} = \lim_{K\to\infty} \frac{\sqrt{K}}{\sqrt{K}+1} = \frac{\infty}{\infty} = 1$

$\sum_{K=1}^{\infty} \frac{1}{\sqrt{K}+1}$ div by L.C.T

Ex: Is $\sum_{n=1}^{\infty} \frac{4n^2+2n}{2n^4+1}$ con or div ?

1) div test: $\lim_{n\to\infty} \frac{4n^2+2n}{2n^4+1} = \frac{\infty}{\infty} \rightsquigarrow$ zero درجة المقام > البسط

divtest failes

2) LCT: geo $\boxed{\times}$ ما بستخدم اختبار التكامل

يشبه ال P sereis ← $n^{\text{قوة عدد}}$

$\sum b_n = \sum \frac{n^2}{n^4}$ (المسيطر) $\frac{\text{أعلى قوة في البسط}}{\text{أعلى قوة في المقام}}$ (نهمل الأرقام)

مع

$= \sum \frac{\cancel{n^2}}{\cancel{n^4}} = \sum_{n=1} \frac{1}{n^2}$ $p = 2 > 1$ conv

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{4n^2+2n}{2n^2+1}}{\frac{1}{n^2}} = 2 > 0$

conv by L.C.T

② $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{\sqrt[2]{n^3}+10}$

1) div test: $\lim_{n\to\infty} \frac{\sqrt{n}+1}{\sqrt[2]{n^3}+10} \Rightarrow \frac{\infty}{\infty} = 0$ درجة المقام أعلى

test failes

2) P-series المقام والبسط بنشيلها

a) L.C.T $= \sum b_n = \sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt[2]{n^3}} = \sum_{n=1}^{\infty} \frac{1}{n}$ $p=1$ div

b) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\frac{\sqrt{n}+1}{\sqrt[2]{n^3}+10}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n(\sqrt{n}+1)}{\sqrt{n^3}+10} = \lim_{n\to\infty} \frac{n^{\frac{3}{2}}+1}{n^{\frac{3}{2}}+10}$

$= 1$

$\sum \frac{\sqrt{n}+1}{\sqrt{n^3}+10}$ div by LCT

Ex: $\sum_{k=1}^{\infty} \frac{1}{3^k+5}$ الـ $\sum$ بتشبه الـ geo أو P series

1) div test: $\lim_{k\to\infty} \frac{1}{3^k+5} = \frac{1}{\infty} = 0$ test failes

$\sum b_k = \sum \frac{1}{3^k} = \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k$ $\boxed{\text{geo}}$ $r = \frac{1}{3} < 1$ conv

$\frac{a_n}{b_n}$ $\lim_{k\to\infty} \frac{a_k}{b_k}$

$\lim_{k\to\infty} \frac{\frac{1}{3^k+5}}{\frac{1}{3^k}} = \lim_{k\to\infty} \frac{3^k}{3^k+5} = \frac{3^k \ln 3}{3^k \ln 3} = 1 > 0$ conv

conv by L.C.T

Ex: $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{8k^2-3k}}$ شبه P

1) div test $= \frac{1}{(8k^2)^{\frac{1}{3}}} = \text{zero}$ k بتقاصر ; مش توزيع من بعلك

$\sum b_k = \sum \frac{1}{k^{\frac{2}{3}}}$ $P = \frac{2}{3} < 1$ div

$\lim_{k\to\infty} \frac{a_k}{b_k} = \frac{\frac{1}{\sqrt[3]{8k^2-3k}}}{\frac{1}{k^{\frac{2}{3}}}} = \lim_{k\to\infty} \frac{k^{\frac{2}{3}}}{\sqrt[3]{8k^2-3k}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2} > 0$ div

0.5 ∛ $\left(\frac{k^2}{8k^2-3k}\right)$ div by L.C.T

Subject: Root test 9 / 1 / 2024

Let $\sum_{k=1}^{\infty} a_k$ be a series :

then:

1) $\lim_{k\to\infty} (a_k)^{\frac{1}{k}} < 1$, $\sum_{k=1}^{\infty} a_k$ conv . متى نستخدم هاد الاختبار؟

• اشهر حالات :

2) $\lim_{k\to\infty} (a_k)^{\frac{1}{k}} > 1$, $\sum_{k=1}^{\infty} a_k$ div . $\cdot \sum n^n$

$\cdot \sum (a_n)^n$

3) $\lim_{k\to\infty} (a_n)^{\frac{1}{k}} = 1$, test failes.

Example :

$\sum_{n=1}^{\infty} n^n$ conv or div ? مش geo لأن الأصل عداد

root test? ومش P

$\lim_{n\to\infty} (n^n)^{\frac{1}{n}} \longrightarrow \lim_{n\to\infty} n = \infty > 1$ div by root test

Examples:

1) $\sum 2^n$ geo ✓

$\lim_{n\to\infty} (2^n)^{\frac{1}{n}} = \lim_{n\to\infty} = 2 > 1$ div أو أي اختبار ثاني ولكن للتوضيح

2) $\sum_{n=1}^{\infty} \left(\frac{2n+1}{8n+1}\right)^n = \lim_{n\to\infty} \left(\left(\frac{2n+1}{8n+1}\right)^n\right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{2n+1}{8n+1} = \frac{1}{4} < 1$

conv by root test

3) $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k \longrightarrow \lim_{k\to\infty} \left(\left(\frac{k}{100}\right)^k\right)^{\frac{1}{k}} = \lim_{k\to\infty} \frac{k}{100} = \infty > 1$ div

by root test

4) $\sum (\tan^{-1} k)^k \rightarrow \lim_{k\to\infty} ((\tan^{-1} k)^k)^{\frac{1}{k}} = \lim_{k\to\infty} \tan^{-1} \infty = \frac{\pi}{2} > 1$ div

by root test

5) $\sum \left(\frac{k}{k+1}\right)^{k^2} \rightarrow \lim_{k\to\infty} \left(\left(\frac{k}{k+1}\right)^{k^2}\right)^{\frac{1}{k}} = \lim_{k\to\infty} \left(\frac{k}{k+1}\right)^k = 1^{\infty}$ غير معرف

Ln ناخذ $y = \left(\frac{k}{k+1}\right)^k \longrightarrow \lim_{y\to\infty} \ln y = \lim \frac{\ln \frac{k}{k+1}}{\frac{1}{k}}$

$a > 1$, $a^{\infty} = \infty$

$1 > a > 0$, $a^{\infty} = \text{zero}$

Subject: root test 14 / 1 / 2023

$$\sum_{K=1}^{\infty} \left(\frac{K}{K+1}\right)^{K^2}$$ con or div ??

لوبيتال:

$\frac{0}{0}, \frac{\infty}{\infty}, 1^{\infty}$

$0^{\infty}, 0\cdot\infty$

$$\lim_{K\to\infty} \left(\left(\frac{K}{K+1}\right)^{K}\right)^{\frac{1}{K}} \to \lim_{K\to\infty} \left(\frac{K}{K+1}\right)^{K} = 1^{\infty}$$

$$\ln y = \ln \left(\frac{K}{K+1}\right)^{K} \rightarrow \ln y = K \ln\left(\frac{K}{K+1}\right)$$

lim للطرفين

$$\lim_{K\to\infty} \ln y = \lim_{K\to\infty} \ln\left(\frac{K}{K+1}\right)$$

(دخلنا لوبيتال ونعمل اشتقاق) $$\lim_{K\to\infty} \frac{\left(\frac{K}{K+1}\right)}{\frac{1}{K}} \rightarrow \lim \frac{\frac{\frac{(K+1)(1)-K}{(K+1)^2}}{\frac{K}{K+1}}}{\frac{-1}{K^2}}$$

$$\lim_{K\to\infty} \frac{\frac{\frac{1}{(K+1)^2}}{\frac{K}{K+1}}}{\frac{-1}{K^2}} \to -\lim_{K\to\infty} \frac{K^2}{(K+1)^2} \cdot \frac{K+1}{K} = \cdot\lim_{K\to\infty} = -1$$

$$\lim_{K\to\infty} \ln y = -1$$

بدي اخلص من ln بحطها e

$$\lim y = e^{-1} = \frac{-1}{e} < 1$$ conv

by root test

Ratio tests:

$n!$

$3! = 3 * 2 * 1$

$n! = n(n-1)!$

$n(n-1)(n-2)!$

فقط اعداد طبيعية مش كسور

ratio tests:

اشهر حالات هنا التست هو!

$\sum a_n$ be a series

① $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$ con

② $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$ div

③ $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$ test fail

Subject:

① Ex: Is $\sum_{n=1}^{\infty} \frac{1}{n!}$ con or div ?

1) $\lim_{n\to\infty} \frac{1}{n!} = \frac{1}{\infty} = 0$ div test failes.

ratio test: $a_n = \frac{1}{n!}$ $a_{n+1} = \frac{1}{(n+1)!}$

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} \right| = \lim_{n\to\infty} \left| \frac{n!}{(n+1)!} \right|$ (ملاحظة لا نلغي ! مع بعضها)

نفكها $(n+1)! = (n+1)n!$

$\lim_{n\to\infty} \frac{\cancel{n!}}{(n+1)\cancel{n!}} = 0 < 1$

conv by ratio test

② Ex: Is $\sum_{k=1}^{\infty} \frac{3^k}{k!}$ → div test failes

$\frac{3^{k+1}}{(k+1)!}$, $\frac{3^k}{k!}$ مكان كل K حط (K+1)

$\lim_{k\to\infty} \frac{3^{k+1}}{(k+1)!} \cdot \frac{k!}{3^k}$ حتى أعرف أحل السؤال لازم أعمل المكتوب

$\lim_{k\to\infty} \frac{\cancel{3^k} \cdot 3 \cdot \cancel{k!}}{(k+1)\cancel{k!} \cdot \cancel{3^k}} = \lim_{k\to\infty} \frac{3}{(k+1)} = 0 < 1$ conv $\sum \frac{k!}{3^k}$ div

* Alternating Series : متناوب

$\sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4 + \ldots$

$\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + \ldots$

Alternating Series conv if: ① $\{a_n\}$ decreasing شروط

② $\lim_{n\to\infty} \{a_n\} = 0$

← continue

Subject: Alternating series test 14 / 1 / 2024

Example: $\sum_{n=1}^{\infty} (-1)^n n$ → a_n 1) $a_n' = 1$ ✗ . اذا أي اشي المهم مايكون صفر في المقام $n = 1$ *

ما دخلها بال limit $(-1)^n$ 2) $\lim \neq 0$ $= \infty$ اذا ما تحقق شرط واحد بكفي

فقط لتحدد Alternat   div seres

Example:

$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ → ① $a_n' = \frac{-1}{n^2} < 0$ ✓

② $\lim_{n \to \infty} \frac{1}{n} = 0$ ✓

conv by Alternating test

Example:

$\sum_{n=1} (-1)^{n+1} \left(\frac{3n}{4n-1}\right)$ → ① $\lim_{n \to \infty} \frac{3n}{4n-1} = \frac{3}{4} \neq 0$

div by Alternating test.

Example ④:

مهم $\sum_{k=1}^{\infty} \frac{\cos(k\pi)}{k}$ $\sin\left(k \frac{\pi}{2}\right)$ مثل   بجرب أعوض عشان أتأكد إذا Alternating أو لا !

$\frac{\cos \pi}{1} + \frac{\cos 2\pi}{2} + \frac{\cos 3\pi}{1} + \dots$ → $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ → conv

$-1 + \frac{1}{2} + \frac{-1}{3} + \frac{1}{4}$

* Absolute conv and contidinally conv : التقارب المطلق والتقارب المشروط

$\sum (-1)^n a_n$ abs conv if $\sum |(-1)^n a_n|$ con

if the series conv and not absolutly conv → conditinally conv

من الشغلات اللي تحل

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$\sum \frac{(-1)^n}{n^3}$ is this series abs conv ?

$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n^3} \right| \rightarrow \sum_{n=1} \frac{1}{n^3}$ ↝ p series

1^n or

abs [illegible]

$P = 3 > 1$ conv

$\sum \frac{(-1)^n}{n^3}$ abs conv

Ex: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ abs conv ??

$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n}$ div

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ not abs conv

الأصلية $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ → $\lim = 0$, $\left(\frac{1}{n}\right) = \text{decr}$ → conv conditionally

Ex: $\sum_{n=1}^{\infty} (-1)^n n$

1) abs ⇒ $\sum_{n=1}^{\infty} \left| (-1)^n n \right| = \sum_{n=1}^{\infty} n$ ↝ div by div test.

2) $\sum (-1)^n n$ not abs conv

conid ?? 3) $\sum (-1)^n n$ → $\lim_{n \to \infty} \neq 0$ → div

اذا كانت abs con مش اكيد con

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$\sum (-1)^n \; a_n$

alternating $\rightarrow \lim_{n\to\infty}$

$\sum a_n$ conv

conditionally

$\sum |(-1)^n a_n| = \text{div}$

$\sum (-1)^n a_n = \text{conv}$

conditionally example:

$\sum (-1)^n \frac{1}{n}$ $\rightarrow \lim_{n\to\infty} \frac{1}{n} = 0$; $\rightarrow \frac{1}{n}$ decreasing ✓ $\Big\}$ conv

$\sum \left| (-1)^n \frac{1}{n} \right| = \sum \frac{1}{n} \rightarrow \text{div}$

conditionally

$\sum \frac{(-1)^n n}{\text{div}}$ $\rightarrow \lim = \infty \neq 0$ div ; $\rightarrow \left| \frac{(-1)^n n}{\text{div}} \right| = \text{div}$