Subject: Sequence   19 / 12 / 2023

* limit of sequence :-

A sequence $\{a_n\}$ is called : 1) convergant sequence if $\lim_{n\to\infty} \{a_n\}$ = number

2) divergant $\lim_{n\to\infty} \{a_n\} = \pm\infty$ D.N.E

Ex: Is the sequence con or div :-

① $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$ $a_n = \{\frac{1}{n}\}$

$\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = \boxed{\text{zero} \to \text{con}}$

② $\{8-4n\}_{n=1}^{\infty}$: $\lim_{n\to\infty} (8-4n) \longrightarrow 8-4(\infty) = \boxed{-\infty \to \text{div}}$

* أي عدد تطرح منه ∞ بتضل ∞

③ $a_n = \{\frac{4n}{6n-1}\}_{n=1}^{\infty}$: $\rightarrow \lim_{n\to\infty} \left(\frac{4n}{6n-1}\right) = \frac{\infty}{\infty}$ لوبيتال $= \frac{4}{6}$ con

④ $a_n = \left\{\frac{7n^3 + 5n^2 + n + 6}{n^2 + 15n^3 + 16}\right\}$: $\lim_{n\to\infty} \frac{7n^3+5n^2+n+6}{n^2+15n^3+16} = \frac{\infty}{\infty}$

بالقانون ① $\frac{\text{المعامل}}{\text{المعامل}}$ درجة البسط = درجة المقام $= \frac{7}{15}$ con

⑤ $a_n = \left\{\frac{n^2+4n+1}{5n-1}\right\}$: $\lim_{n\to\infty} \left\{\frac{n^2+4n+1}{5n-1}\right\}$ لوبيتال درجة البسط أعلى ما بنفع $\frac{\text{معامل}}{\text{معامل}}$

درجة البسط أعلى من المقام ← حل النهاية ذات $\pm\infty$ $\frac{2n+4}{5} = \infty$ div

6) $a_n = \left\{\frac{n^2}{-n}\right\} \longrightarrow \frac{2n}{-1} \longrightarrow -\infty$ div

7) $a_n = \left\{\frac{\ln n}{n}\right\}_{n=1}^{\infty} \longrightarrow \lim_{n\to\infty} \frac{\ln n}{n} \longrightarrow \frac{\ln\infty}{\infty} = \frac{\infty}{\infty}$ لوبيتال

$\lim_{n\to\infty} \frac{\frac{1}{n}}{1} = \frac{1}{\infty}$ = zero con

8) $a_n = \{\ln(2n+1) - \ln(n+2)\}$

$\lim_{n\to\infty} \{\ln(2n+1) - \ln(n+2)\} = \infty - \infty$!!! مستحيل

$\lim_{n\to\infty} \ln\frac{(2n+1)}{(n+2)} \rightarrow \ln\left(\lim_{n\to\infty} \frac{2n+1}{n+2}\right) = \ln 2$ con $\frac{\text{معامل}}{\text{معامل}}$

⑨ $a_n = \{\ln(n^2+n+1) - \ln(n+8)\}$

$\lim_{n\to\infty} \{\ln(n^2+n+1) - \ln(n+8)\}$ * التعويض المباشر $\infty - \infty$

$\lim_{n\to\infty} \ln\left(\frac{n^2+n+1}{n+8}\right)$ * لو كانت بينهم + ← $\infty+\infty$ خلص بوقف

$\ln\left(\lim_{n\to\infty}\left(\frac{n^2+n+1}{n+8}\right)\right) \xrightarrow{\text{اشتق}} \lim_{n\to\infty}\left(\frac{2n+1}{1}\right) \to \infty$ div

⑩ $a_n = \sin^{-1}\left(\frac{n+8}{2n+17}\right)_{n=1}^{\infty} \to \lim_{n\to\infty} \sin^{-1}\left(\frac{n+8}{2n+17}\right) \to \sin^{-1}\left(\lim \frac{n+8}{2n+17}\right)$

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ con

⑪ $\{(-1)^n\}_{n=1}^{\infty} = \{-1, 1, -1, 1, \ldots\}$

sequence جزئية odd $\to -1, -1, -1, \ldots \to -1$ → dive

Subsequence even $\to 1, 1, 1, \ldots \to 1$

Example: is the seq $\{\underset{n=1}{1}, \underset{n=2}{\frac{1}{2}}, \underset{n=3}{3}, \underset{n=4}{\frac{1}{4}}, 5, \frac{1}{6}, \ldots\}$ con or div

subsequence $n \to$ odd : n $\left(\lim_{n\to\infty} n = \infty\right)$

subsequence $n \to$ even : $\frac{1}{n}$ $\left(\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0\right)$ → div

* Monotone sequence:

مثل Example: $\{1, \frac{-1}{2}, \frac{1}{3}, \frac{-1}{4}, \frac{1}{5}, \ldots\}$ conv

* Monotone sequence:

① A sequence $\{a_n\}_{n=1}^{\infty}$ is called increasing sequence if $a_1 \leq a_2 \leq a_3 \leq \cdots \leq a_n \leq \cdots$ or $(a_n' > 0)$ for all n.

② A sequence $\{a_n\}_{n=1}^{\infty}$ is called decreasing sequence if $a_1 \geq a_2 \geq a_3 \geq \cdots$ or $(a_n' < 0)$

③ A sequence $\{a_n\}_{n=1}^{\infty}$ is called monotone sequence if it is increasing $\boxed{\text{or}}$ decreasing.

Example:

1) A seq $\{n^2\}_{n=1}^{\infty}$

$\{1, 4, 9, 16, 25 \dots\}$

$1 < 4 < 9 < 16 < 25 < \dots \rightarrow$ increasing sequence

OR: $a_n = n^2$

$a'_n = 2n > 0 \rightarrow$ monotone sequence.

2) $a_n = \{\frac{1}{n}\}_{n=1}^{\infty}$

$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6} \dots\}$

$1 > \frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \dots \rightarrow$ decreasing sequence

or $a'_n = \frac{-1}{n^2} < 0 \rightarrow$ monotone sequence.

3) $\{-1, 1, -1, 1, \dots\}$

$-1 < 1 > -1 < 1 > -1 \rightarrow$ not increasing and not decreasing.

not monotone.

4) $a_n = n e^{-n} \rightarrow \frac{n}{e^n}$

$a'_n = \frac{(e^n)(1) - n e^n}{(e^n)^2} = \frac{e^n(1-n)}{e^{2n}} \rightarrow \frac{1-n}{e^n} < 0$

decreasing sequence

monotone sequence

2) $a_n = \tan^{-1} n \rightarrow a'_n \rightarrow \frac{1}{1+n^2} > 0$ increasing sequence

monotone sequence

3) $a_n = \frac{n}{n+1}$

$a'_n = \frac{(n+1)(1) - n(1)}{(n+1)^2} = \frac{1}{(n+1)^2} > 0$ increasing sequence

7) $a_n = (n-7)^2$

$a'_n = 2(n-7) \rightarrow$ not monotone

8) $a_n = (\sin^{-1} n)_{n=1}^{\infty}$ X

8) $a_n = \{9, 9, 9, 9, \dots\}$ constant

$9 \geq 9$ $a_n = \{9\}$

في مساواة .

* Bounded sequence : Seq محدودة

1) a sequence a_n is called Bounded above. (if $a_n \leq M$ for all n) مح من فوق

2) a sequence a_n is called Bounded below (if $a_n \geq m$ for all n) مح من تحت

3) a sequence a_n is called Bounded below and Bounded above.

example:

1) $a_n = \{n^2\}_{n=1}^{n=\infty}$

$\{1, 4, 9, 16, 25, 36, 49, \dots\}$

not Bounded above مافي رقم كل العناصر أكبر منه

Bounded below فيه رقم أصغر منهم كلهم (1) . مش Bdd بشكل عام لأنه ما حققت الشرطين

2) $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots\}$ $\{\frac{1}{n}\}^{\infty}$

bdd above 1 اكبر منهم كلهم

bdd below → (bdd) مثلا 0 و -1 اصغر منهم كلهم

3) $\{-1, 1, -1, 1, -1, \dots\}$ bdd below في عدد اكبر منهم كلهم

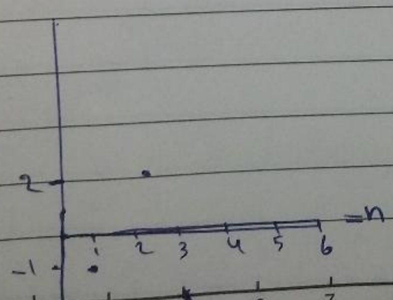
bdd above . في عدد اقل منهم كلهم

Example:

$$a_n = \begin{cases} n & : n \text{ even} \\ -n & : n \text{ odd} \end{cases}$$

$\{-1, 2, -3, 4, -5, 6, -7, 8, \dots\}$ final مثال

not bdd



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Theorem:

1) every bdd & sequence monotone is conv.

Example: consider $\{\frac{1}{n}\}_{n=1}^{\infty}$

$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\}$ 1) bdd → below مقسوم <1
→ above مقسوم >

2) monotone = $\frac{-1}{n^2} < 0$ (mono, decreasing)

conv

* bdd $\not\to$ conv — الـ bdd مش ضروري تكون conv

$a_n \to \{-1\}^n = \{-1, 1, -1, 1, \dots \infty\}$ bdd

not conv

* mono $\not\to$ — الـ bdd و mono يعني con

a_n $\{n\} = \{1, 2, 3, 4, 5, \dots\}$

$a_n' = 1 > 0$

$\lim_{n\to\infty} n = \infty$ div

div seq:

not bdd or not monotone or both of them

Ex: a_n $(-1)^n = \{-1, 1, -1, 1, \dots\}$

div since not monotone.

$a_n = n^2 = \{1, 4, 9, 16, 25, \dots\}$ mono ✓

div since not bdd

* Series: المتسلسلات — 12-15 الساعة ماتيل

Let $\{a_n\}_{n=1}^{\infty}$ be a seq

مجموع $S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$S_4 = a_1 + a_2 + a_3 + a_4$

$S_n = a_1 + a_2 + \dots a_n = \sum_{i=1}^{n} a_i$

The seq $\{S_n\}_{n=1}^{\infty}$ is called seq of partial sums

متتالية

مجاميع جزئية

Bim

← con

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$\{S_n\}$ conv if $\lim\limits_{n\to\infty} S_n = \lim \sum\limits_{i=1}^{n} a_i$ conv (number)

$= \sum\limits_{i=1}^{\infty} a_i$

* $S_n \to$ conv $\rightsquigarrow \sum\limits_{i=1}^{\infty} a_i$ conv.

$S_n \to$ div $\rightsquigarrow \sum\limits_{i=1}^{\infty} a_i$ div.

Example:

let $\{a_n\}^{\infty}$ be a seq with $S_n \left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$

$S_1 = a_1$

$S_2 = a_1 + a_2$

is $\sum\limits_{i=1}^{\infty} a_i$ conv or div ?

Sol: $\lim\limits_{n\to\infty} S_n = \sum\limits_{i=1}^{\infty} a_i \to \lim\limits_{n\to\infty} S_n \to \lim\limits_{n\to\infty} \frac{n}{n+1} = \frac{\infty}{\infty} \longrightarrow$ لوبيتال

$1 \to$ conv

Example:

$\{a_i\}_{i=1}^{\infty}$ be a seq with $S_n = \{e^n\}_{n=1}^{\infty}$, is $\sum\limits_{i=1}^{\infty} a_i$ div or con ??

$\sum\limits_{i=1} a_i$ $\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} e^n = e^{\infty} = \infty$

$\sum\limits_{i=1}^{\infty} a_i$ div series

Example:

Is $\sum\limits_{i=1}^{\infty} \underbrace{\left(\frac{1}{i} - \frac{1}{i+1}\right)}_{a_i / a_n}$ con or div

note:

$\lim\limits_{n\to\infty} a =$

$a > 1 = \infty$

$0 < a < 1 = 0$

$a_i = \left(\frac{1}{i} - \frac{1}{i+1}\right)$

$S_1 = a_1 \longrightarrow \left(1 - \frac{1}{2}\right) = \frac{1}{2}$

$S_2 = a_1 + a_2$

$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) =$

$S_3 = a_1 + a_2 + a_3 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right)$

$S_n = a_1 + a_2 + a_3 + \dots \underset{\text{قبل}}{a_{n-1}} + a_n$ هون بتختصر

$S_n = \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{n-1} - \frac{1}{n}\right) + \left(\frac{1}{n} - \frac{1}{n+1}\right)$

هي اللي بعدها هي اللي قبلها

$S_n = 1 - \frac{1}{n+1}$ ضل اول حد وآخر حد

$\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = 1$ conv

$\sum \frac{1}{i} - \frac{1}{i+1}$ conv

Ex: is $\sum_{i=1}^{\infty} [\underbrace{\tan^{-1}(i) - \tan^{-1}(i+1)}_{a_i}]$ con or div ?

ا. بطلع S_n

$S_1 = a_1$

$\tan^{-1}(1) - \tan^{-1}(2)$

$S_n = a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n$

$(\tan^{-1}(1) - \cancel{\tan^{-1}(2)}) + (\cancel{\tan^{-1}(2)} - \cancel{\tan^{-1}(3)}) + (\cancel{\tan^{-1}(3)} - \cancel{\tan^{-1}(4)}) + \dots + (\cancel{\tan^{-1}(n-1)} - \cancel{\tan^{-1}(n)}) +$
$(\cancel{\tan^{-1}(n)} - \tan^{-1}(n+1))$

$S_n = \tan^{-1}(1) - \tan^{-1}(n+1)$

$\lim_{n\to\infty} S_n = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{2}$ conv

Recall Example:

① $\sum_{n=1}^{\infty} \underbrace{\left(\frac{1}{n} - \frac{1}{n+1}\right)}_{a_n}$ con or div ?

$S_1 = a_1 \rightarrow \frac{1}{1} - \frac{1}{2}$

$S_n = (\ ) + (\ ) + \dots + (\ ) + (\ )$

$\tan^{-1}(n+1)$ $-\tan^{-1}(n)$ $+\tan^{-1}(n)$ $-\tan^{-1}(10)$

② $\sum_{i=1}^{\infty} \underbrace{(\tan^{-1}(i+2) - \tan^{-1}(i))}_{a_n \text{ or } a_i}$ conv or div ? $\tan^{-1}(n-2)$

$S_n = a_1 + a_2 + a_3 + \dots + a_{(n-2)} + a_{(n-1)} + a_{(n)}$

$S_n = \underbrace{\cancel{\tan^{-1}(3)} - \tan^{-1}(1)}_{a_1} + \underbrace{\cancel{\tan^{-1}(4)} - \tan^{-1}(2)}_{a_2} + \underbrace{\tan^{-1}(5) - \cancel{\tan^{-1}(3)}}_{a_3} + \cancel{\tan^{-1}(n)} - \tan^{-1}(n-2)$ (n-4)

$+ \tan^{-1}(n+1) - \cancel{\tan^{-1}(n-1)} + \tan^{-1}(n+2) - \cancel{\tan^{-1}(n)}$

$-\tan^{-1}(1) - \tan^{-1}(2) + \tan^{-1}(n+1) + \tan^{-1}(n+2)$

$\lim_{n\to\infty} S_n = \underbrace{-\tan^{-1}(1) = \frac{\pi}{4} - \tan^{-1}(2)}_{\text{ثوابت}}$

$\lim_{n\to\infty} \tan^{-1}(n+1) \longrightarrow \frac{\pi}{2}$

$\frac{\pi}{4} - \tan^{-1}(2) + \frac{\pi}{2} + \frac{\pi}{2}$

con

$\lim_{n\to\infty} \tan^{-1}(n+2) \longrightarrow \frac{\pi}{2}$

Sunday 31 / dece /

Is the following con or div :-

① $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ → by partial fraction

$S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n(n+1)}$

① $\frac{1}{n(n+1)} = \frac{A}{(n)} + \frac{B}{(n+1)} \rightsquigarrow 1 = A(n+1) + B(n)$

$n=0 \rightarrow A = 1$

$n=-1 \rightarrow B = -1$

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} \quad a_n = \sum \left(\frac{1}{n} - \frac{1}{(n+1)}\right)$ continue !

② $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=1}^{\infty} \ln(n) - \ln(n+1)$ continue

$\ln(1) - \ln(\infty)$

③ $\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}} - e^{\frac{1}{n+1}}\right) \rightarrow S_n = a_1 + a_2 +$

$(e^{1} - e^{\frac{1}{2}}) + (e^{\frac{1}{2}} - e^{\frac{1}{3}}) + (e^{\frac{1}{3}} - e^{\frac{1}{4}}) + (e^{\frac{1}{n-1}} - e^{\frac{1}{n}}) + (e^{\frac{1}{n}} - e^{\frac{1}{n+1}})$

$S_n = e^{1} - e^{\frac{1}{n+1}}$

$\lim_{n \to \infty} S_n = e^{1} - e^{\frac{1}{\infty}} \rightarrow e - 1 \rightarrow$ conv

* Geometric series المتسلسلات الهندسية

$\sum_{n=0}^{\infty} a r^{n}$ القوة في العداد

Ex: $\sum_{n=0}^{\infty} 7(3)^{n}$ $a \to 7$ geometric

$r = 3$

$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k}$ $a = 1$ geometric

$r = \frac{1}{2}$

$\sum_{n=5} n^{2}$ not geometric

* $\sum_{n=1}^{\infty} \ln(n)^{\alpha}$ not geometric

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$\sum_{k=0}^{\infty} k^k$ not geometric

$\sum_{k=2}^{\infty} \frac{3^k}{4^k} \rightarrow \sum_{k=2}^{\infty}\left(\frac{3}{4}\right)^k$ geometric

$a=1$

$r=\frac{3}{4}$

A geometric series is con:

$\sum_{n=0}^{\infty} ar^n \rightarrow$ conv $-1<r<1$

$\rightarrow$ di $r<-1$ or $r>1$

example:

1) $\sum_{n=0}^{\infty} ar^n$ if conv

$= \frac{\text{first term}}{1-r}$

Is the following is conv or div?

1) $\sum_{n=0}^{\infty} 2^n$ geo $\boxed{\checkmark}$

$r=2 \rightarrow$ div

2) $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ geo $\boxed{\checkmark}$

$r=\frac{1}{2}<1$ conv

$= \frac{\left(\frac{1}{2}\right)^0}{1-\frac{1}{2}} \longrightarrow \frac{1}{1-\frac{1}{2}} = \boxed{2}$

$\Sigma = 1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\dots = 2$

3) $\sum_{k=1}^{\infty} 7\left(\frac{1}{3}\right)^k$

1) geo $\boxed{\checkmark}$

2) $r=\frac{1}{3}<1$ conv

3) Sum $= \frac{7\left(\frac{1}{3}\right)^1}{1-\frac{1}{3}} = \frac{7}{2}$

4) $\sum_{k=0}^{\infty} 10\,2^k \cdot 9^{-k} \rightarrow \sum_{k=0}^{\infty} \frac{10\,2^k}{9^k} \rightarrow 10\left(\frac{2}{9}\right)^k$ geo $\boxed{\checkmark}$

$r=\frac{2}{9}<1$ con

Sum $= \frac{10}{1-\frac{2}{9}} = \frac{90}{7}$

Subject:

5) $\sum_{n=0}^{\infty} 3^{\frac{-n}{2}} = \sum_{n=0}^{\infty} \frac{1}{3^{\frac{n}{2}}} = \sum_{n=0}^{\infty} \frac{1}{((3)^{\frac{1}{2}})^n} = \sum_{n=0}^{\infty} = \left(\frac{1}{\sqrt{3}}\right)^n$ geo ✓

$r = \frac{1}{\sqrt{3}} < 1$ con

Example: is $\sum_{K=2}^{\infty} \frac{(-1)^K}{3^K}$ conv or div ?

$|r| < 1 \rightarrow$ con

$\sum_{K=2}^{\infty} \left(\frac{-1}{3}\right)^K$ geometric ✓

$r = \frac{-1}{3} < 1$ conv

طريقة ثانية

$\left|\frac{-1}{3}\right| < 1$ conv

$Sum = \frac{\left(\frac{-1}{3}\right)^2}{1-\left(\frac{-1}{3}\right)} = \frac{1}{12}$

② is $\sum_{K=1}^{\infty} 3^{2K} 5^{1-K}$ con or div ?

$(3^2)^K \rightarrow 9^K$ $\quad 5^1 \times 5^{-K}$

$\sum_{K=1}^{\infty} \frac{9^K 5^1}{5^K} \rightarrow \sum_{K=1}^{\infty} 5 \left(\frac{9}{5}\right)^K \rightarrow a = 5$

$r = \frac{9}{5} > 1$ div {sum ما في div}

③ $\sum_{K=1}^{\infty} \left(\frac{1}{2^K} - \frac{1}{2^{K+1}}\right)$ determine whether it's con or div ?

توحيد مقامات $\sum_{K=1}^{\infty} \left(\frac{1 \times 1}{2 \times 2^K} - \frac{1}{2^{K+1}}\right) = \frac{2-1}{2^{K+1}} = \sum_{K=1}^{\infty} \frac{1}{2^K} \cdot \frac{1}{2} \rightarrow \sum_{K=1}^{\infty} \frac{1}{2} \cdot \left(\frac{1}{2}\right)^K$

$r < 1 \rightarrow$ con

$Sum = \frac{\frac{1}{4}}{1-\frac{1}{2}} =$

Ex: write as a fraction

④ 0.4444.....

$= 0.4 + 0.04 + 0.004 + 0.0004 + \ldots$ geo ✓

قسمة الحد على الحد اللي قبله $r = \frac{0.04}{0.4} = \frac{0.004}{0.04} = 0.1$

مين إيلها على شكل كسر

$\sum_{K=1}^{\infty} 4 \underbrace{(0.1)}_{r}^K = 0.4 \rightarrow \frac{\frac{4}{10}}{1-\frac{1}{10}} = \frac{4}{9}$ con

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Ex:

(4) 0.373737 .... (as fraction) اللي بتكرر 37 مش 7 لحالها و 3 لحالها

$= 0.37 + 0.0037 + 0.000037 + \dots\dots$

فبيصير $r = 0.01$

$\sum_{k=1}^{\infty} 37(0.01)^k \quad < 1 \qquad$ Sum =

(5) 5.373737 .....

نرحلها للأخير $= 5 + 0.373737\dots\dots$

$5 + \sum_{k=1}^{\infty} 37\,(0.01)^r$

(6) Show that $\sum_{k=0}^{\infty} (-1)^k (x)^k = \frac{1}{1+x}$, $-1 < x < 1 \longrightarrow$ conv

geo $\boxed{r}$

تعويض k $\quad$ L.H.S $= 1 + -x + x^2 - x^3 + \dots\dots$

$\sum_{k=0}^{\infty} (-x)^k$ conv, geo

$\boxed{-x^0 = 1 \quad k=0} \longleftarrow$ sum $\frac{1}{1-r} = \frac{1}{1+x}$ #

* Convegence tests:

[1] divergent tests:

let $\sum a_n$ be a series then (1) if $\lim_{n\to\infty} a_n \neq 0 \to \sum a_n$ div

(2) if $\lim_{n\to\infty} = 0 \to$ test fail $\left\langle \begin{array}{l} \text{con} \\ \text{div} \end{array}\right.$

Ex:

(1) $\sum_{k=0}^{\infty} 2^k \longrightarrow$ geo $\boxed{r}$ $\quad r > 1$ div

$\lim_{k\to\infty} 2^k = 2^\infty = \infty \neq 0$

div by div test

2) $\sum_{n=1}^{\infty} \frac{3n+1}{4n-2}$ geo X الأساس مش ثابت

$\lim_{n\to\infty} \frac{3n+1}{4n-2} = \frac{\infty}{\infty} = \frac{3}{4} \neq 0$

div by div test

③ $\sum_{n=2}^{\infty} \frac{n}{\ln n} \rightarrow \lim_{n\to\infty} \frac{n}{\ln n} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{1}{\frac{1}{n}} \rightarrow \lim_{n\to\infty} n = \infty \neq 0$

div by div test

④ $\sum_{K=1}^{\infty} \tan^{-1} K$ geo $\boxtimes$

$\lim_{K\to\infty} \tan^{-1} K \longrightarrow \tan^{-1}\infty = \frac{\pi}{2} \neq 0$

div by div test

⑤ $\sum_{n=1}^{\infty} \frac{1}{n}$ geo $\boxtimes$

$\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$ test failes. div آخر test

⑥ $\sum_{n=1}^{\infty} \frac{1}{n^2}$ geo $\boxtimes$

$\lim_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty} = 0$ test failes. con آخر test

② integral test

If Σa_n be a series

Σa_n conv $\rightarrow \int a_n$ con

Σa_n div $\rightarrow \int a_n$ div

$\sum_{K=1}^{\infty} a_n \quad \int_1^{\infty} a_n \, dn \rightarrow$ impropar < conv / div

→ 1) integral
→ 2) subtitute
→ 3) lim

Σa_n be a series ① positive
② decreasing
③ cont

Integral test:

$$\sum_{n=1}^{\infty} a = \int_1^{\infty} a_n \, dn$$

① $a_n \geq 0$

② decreasing

③ cont ((لا يوجد أصفار مقام، جذر سالب، (منفصل))) متصل