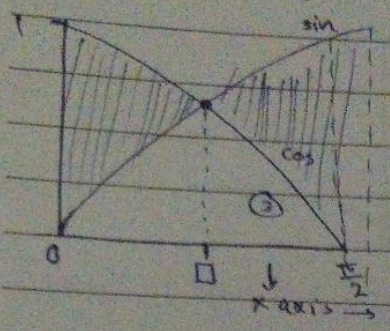
Subject: / /

← Example شرح الحل



$$\int_0^{\frac{\pi}{4}} (\cos(x) - \sin(x))\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin(x) - \cos(x))\,dx$$

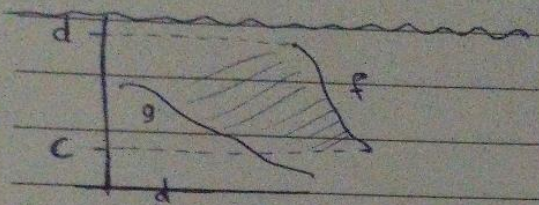
هنا اوجد النقطة بساوي الاقترانين بعض

$\sin x = \cos x = \frac{1}{\sqrt{2}}$

او قسمة $\frac{\sin x}{\cos x} = 1$ $x = \frac{\pi}{4}$

$\tan x = 1 \rightarrow x = \frac{\pi}{4}$

لو طلب السؤال بين الاقترانين والمحور x اوجد المساحة الثالثة

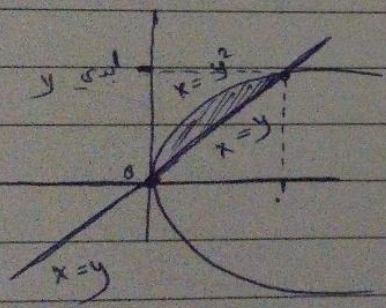


$x = g(y)$ بدلالة y

$x = f(y)$

$$A = \int_c^d f(y) - g(y)\,dy$$

Example: find the area between $x = y^2$ and $x = y$



بساوي y:

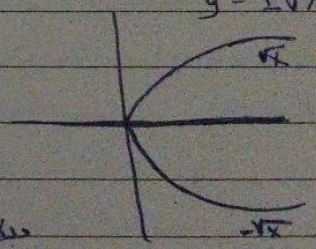
$y^2 = y$

$y^2 - y = 0$

$y = 0 , y = 1$

شرح رسمة $x = y^2$

$y = \pm\sqrt{x}$

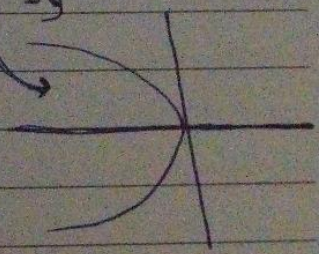


بدلالة y

البعيد عن المحور y - القريب على المحور y

$$A = \int_0^1 (y - y^2)\,dy$$

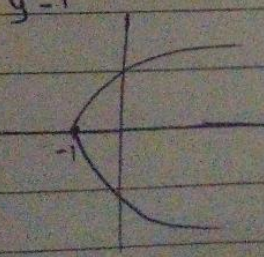
$x = -y^2$



بدلالة x

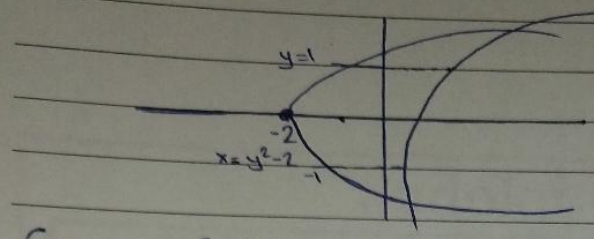
الفوق - التحت

$x = y^2 - 1$



Subject: Area — Tuesday — 28 / November / 2023

Example: find the area $x = y^2 - 2$, $x = e^y$ from $y = -1$ to $y = 1$:-



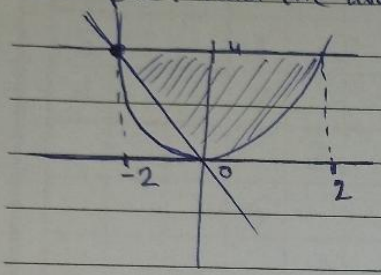
$$A = \int_{-1}^{1} e^y - (y^2 - 2)\, dy$$

$x = e^y$

$\ln x = \ln e^y$

$y = \ln x$

Example: find the area $y = x^2$, $y = 4$, $y = -2x$

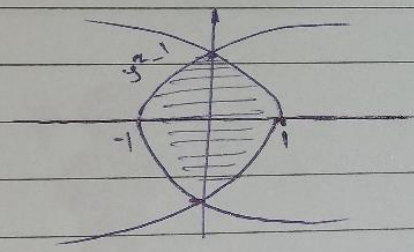


$x^2 = 4 \rightarrow x = \pm 2$

المنحنيات تتقاطع عند $y = 4$

$$A = \int_{-2}^{0} (4 - (-2x))\, dx + \int_{0}^{2} (4 - x^2)\, dx$$

Example: find the area $x = y^2 - 1$, $x = 1 - y^2$:-



$$A = \int_{-1}^{1} (1 - y^2) - (y^2 - 1)\, dy$$

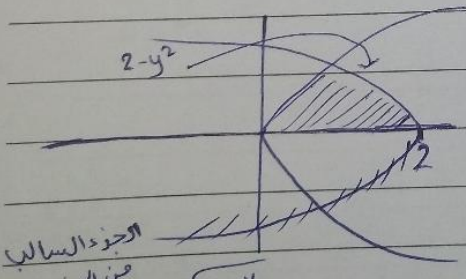
حدود التكامل

$1 - y^2 = y^2 - 1$

$y = \pm 1$

Example: find the area $x = y^4$, $y = \sqrt{2 - x}$, $y = 0$.

$y = \sqrt[4]{x}$



$$A = \int_{0}^{1} (2 - y^2) - y^4\, dy$$

نعم $\sqrt{x}$ زي $\sqrt[4]{x}$

بس أخليهم كلهم بدلالة x

$y = \sqrt{2 - x}$

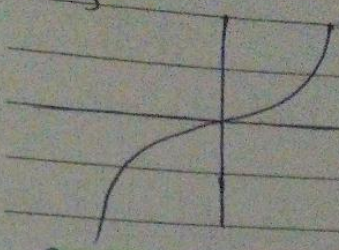
$x = 2 - y^2$

← area

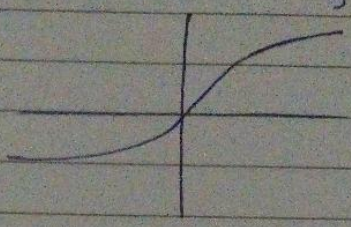
Subject:

28 / 11 / 2023

$y = x^3, x^5, x^7$



$y = \sqrt[3]{x}$



✱ How to integrate:

① table (simplify) → $\int \frac{(x^4-1)}{(x^2+1)}\,dx \rightarrow \int \frac{(x^2-1)(x^2+1)}{(x^2+1)}\,dx$

② substitution → $\int \frac{1}{[u]}$, $\int \sin [u]$, $\int e^{[u]}$, $\int ([u])$.

$\int x^3 \sqrt{x^4+1}\,dx$, $\int 2^x \sin 2^x$

③ Trig integ → $\int \sin(x)dx$. $\int \cos x\,dx$. $\int \cos^2(dx)$. -

✱) Rational function → $\int \frac{P(x)}{Q(x)}$

5) by parts → $\int$ (Poly) (trig, exp, log) o

$\int \sin^{-1} dx$, $\int \tan^{-1} dx$, $\int \log x\,dx$

6) $\sqrt{x^2+a^2} \Rightarrow x = a\tan\theta$, $\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta$, $\sqrt{x^2-a^2} \rightarrow x = a\sec\theta$

7) $\int e^{\sqrt{x}}\,dx \rightarrow u = \sqrt{x} \rightarrow \int e^u \cdot 2u\,du \rightarrow$ by parts

$\int \frac{x^2}{x^6+3x^3+2}\,dx \rightarrow \int \frac{x^2}{(x^3)^2+3x^2+2}$

$u = x^3$

$du = 3x^2 dx$

$dx = \frac{du}{3x^2}$

$\int \frac{x^2}{u^2+3u+2} \times \frac{du}{3x^2} \rightarrow \frac{1}{3}\int \frac{1}{(u+1)(u+2)}\,du$

← combine

Subject: ____ / /

② $\int \frac{x e^x}{(1+x)^2} dx \rightarrow \int x e^x (1+x)^{-2} dx$

$= x e^x (1+x) + \int e^x dx \rightarrow x e^x (1+x) + e^x + C$

$u = x e^x$

$du = x e^x + e^x$

$v = \frac{(1+x)^{-1}}{-1}$

③ $\int \frac{\ln(\tan x)}{\sin x \cos x}$

$u = \ln \tan(x)$

$du = \frac{\sec^2(x)}{\tan}$ or $\frac{\frac{1}{\cos^2}}{\frac{\sin}{\cos}}$

$= \frac{1}{\sin \cdot \cos}$

④ $\int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx = \int \frac{1}{x^{\frac{1}{2}} - x^{\frac{1}{3}}} dx$

$= \int \frac{1}{u^3 - u^2}$

$u = \sqrt[6]{x}$

$(u) = (x^{\frac{1}{6}})^2 \rightarrow u^2 = x^{\frac{1}{3}}$

$du = \frac{1}{6} x^{1-\frac{1}{6}}$

$du = \frac{1}{6} x^{\frac{-5}{6}}$

$\frac{1}{6} \times \frac{1}{x^{\frac{5}{6}}} \rightarrow \frac{1}{6} \cdot \frac{1}{u^5}$

⑤ $\int x \tan^2(x) dx$

$u = x \quad du = 1$

$dv = \tan^2(x) \quad v = \tan x - x$

⑥ $\int \frac{\sqrt{x}}{\sqrt[3]{x} - \sqrt[5]{x}} dx$

$u = x^{\frac{1}{30}} \quad u^{30} = x$

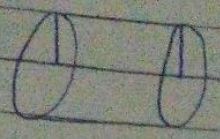
$u^{\frac{30}{15}} = x^{\frac{1}{2}}$

$u^{10} = x^{\frac{1}{3}}$

$\int e^u \; 2u \, du$

Subject: Volume الحجوم الدورانية Sunday 10 / december 2023

1)

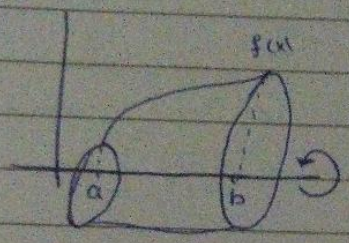


$V = \pi r^2 h$

2)

$V = \frac{1}{3}\pi r^2 h$

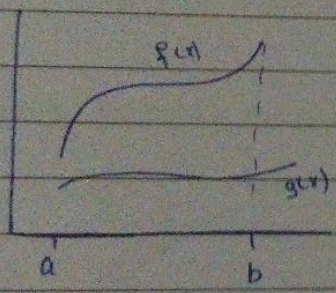
3)



when we rotate f(x) about x axis from a=x to x=b

$$V = \pi \int_a^b (f(x))^2 \, dx$$

4)

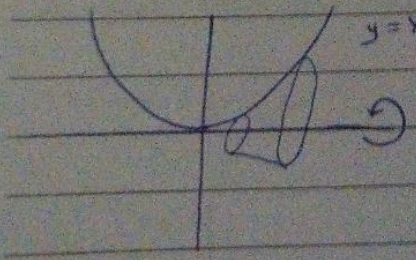


$$V = \pi \int_b^a f(x)^2 - g(x)^2 \, dx$$

rotate about x axis

1) example: find the volume when we rotate the area $y = x^2$ from $x = 1$ to $x = 3$ about x-axis

في السؤال 3 معطيات:
1- قترانات 2- حدود التكامل 3- محور الدوران



تدور الاقتران بين حدود التكامل

$$V = \pi \int_1^3 x^4 \, dx = \pi \frac{x^5}{5} \Big|_1^3 = \pi \left(\frac{3^5}{5} - \frac{1^5}{5}\right)$$

$\frac{242\pi}{5}$ مقدار الحجم

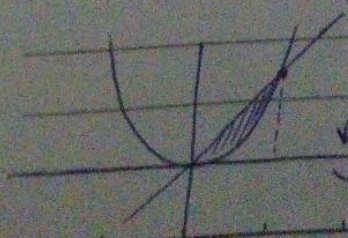
2) example find the volume when we rotate the area between $y = x$ و $y = x^2$ about the x-axis

نقاط تقاطع الاقترانات $x = x^2$

$x = 0$

$x = 1$

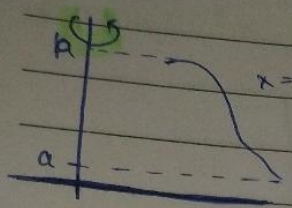
$$V = \pi \int_0^1 [x^2 - x^4] \, dx$$



$\frac{2\pi}{15}$ مقدار الحجم

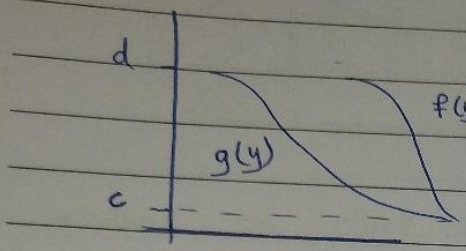
Bim

Sunday 10 / 12 / 2023



about y axis

$$V = \pi \int_a^b (g(y))^2 \, dy$$



$$V = \pi \int_c^d f(y)^2 - g(y)^2 \, dy$$

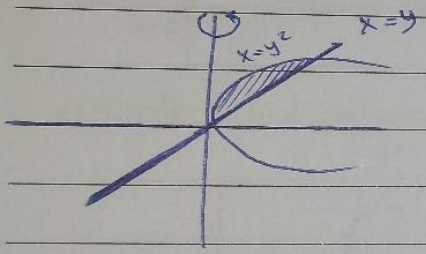
example: find the volume when we rotate $x = \sqrt[3]{y}$ from $y=1$ to $y=3$ about y axis

$$V = \pi \int_1^3 (\sqrt[3]{y})^2 \, dy \longrightarrow V = \pi \int_1^3 y^{\frac{2}{3}} \, dy$$

$$V = \pi \left. \frac{y^{\frac{2}{3}+1}}{\frac{2}{3}+1} \right|_1^3 = \frac{3\pi}{5}(\sqrt[3]{243})$$

example: find the volume when we rotate $x=y$, $x=y^2$ about y-axis

حدود التكامل من نقاط تقاطع الاقترانات



$$V = \pi \int_0^1 y^2 - y^4 \, dy$$

$y=0$

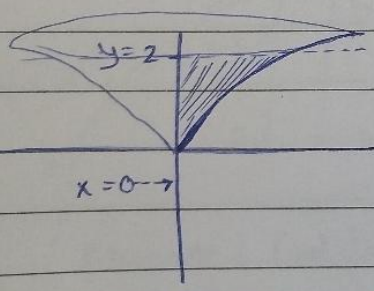
$y=1$

كل اقتران تربيع لحال

$$\left. \frac{y^3}{3} - \frac{y^5}{5} \right|_0^1$$

بنفس مثال الكتاب $\frac{2\pi}{15}$

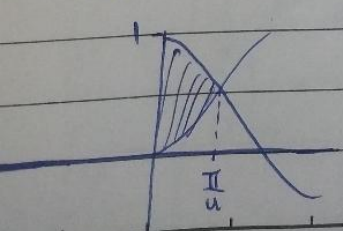
example: find the volume when we rotate $y = \sqrt{x}$ ($y^2 = x$), $y=2$ and $x=0$ about y-axis



$$V = \pi \int_0^2 (y^2)^2 - (0 \leftarrow \text{y axis}) \, dy$$

$\frac{32\pi}{5}$

example: find the volume when we rotate $y = \cos(x)$, $y = \sin(x)$ $x = 0 \to \frac{\pi}{4}$



$$V = \pi \int_0^{\frac{\pi}{4}} \cos(x)^2 - \sin^2(x) \, dx$$

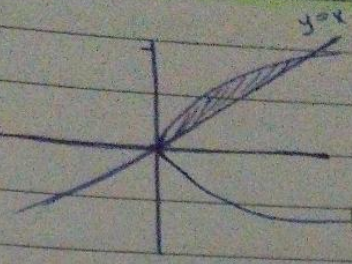
about x-axis

$\int \cos 2x$

$\frac{\pi}{2}$

10/ 12 / 2023

example: find v when we rotate the area between $\sqrt[4]{x} = y$, $y = x$ about y-axis.



$$v = \pi \int_0^1 y^2 - (y^4)^2$$

$$= \frac{4\pi}{21}$$

بطلوع بدلالة y

$\sqrt[4]{x} = y$

$x = y^4$

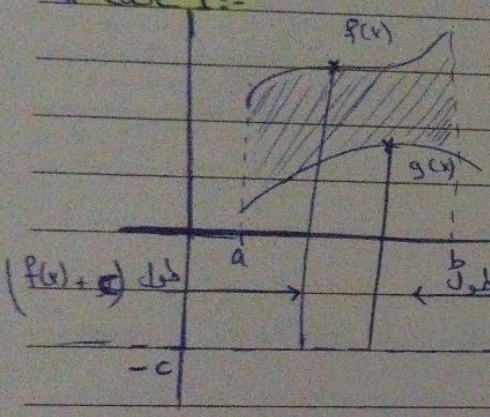
$y^4 = y$

$y^4 - y = \text{zero}$

$y = 0, 1$

* volume on other axis of revolution :-

* case 1 :-



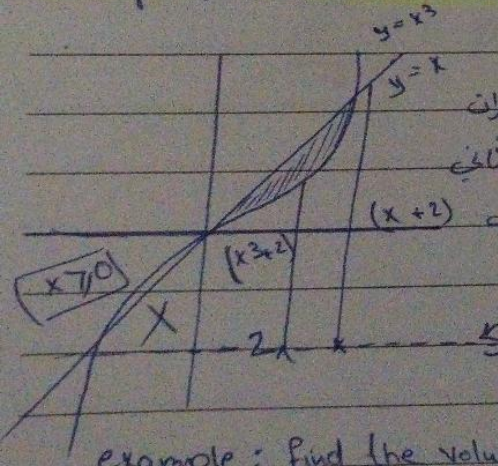
volume when we rotate the area between $y = f(x)$, $y = g(x)$ from $x = a$ to $x = b$ about $y = -c$.

* لما يكون دوران حول x ولا y يوصل خطين

$$v = \pi \int_a^b (f(x)+c)^2 - (g(x)+c)^2 \, dx$$

(الكبير - الصغير)

example : find volume when we rotate the area between $y = x^3$, $y = x$, $x \geq 0$ about $y = -2$



* خطين من دوران الاقتران الأول والثاني

* الطول دائما موجب

حدود التكامل = $x^3 - x = 0$

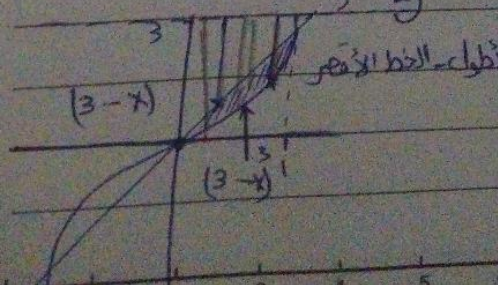
$x(x^2 - 1) = x = 0$

$x = \pm 1 \quad (x \geq 0)$

$$v = \pi \int_0^1 (x+2)^2 - (x^3+2)^2 \, dx$$

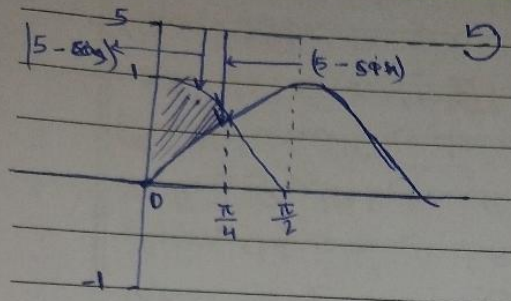
example : find the volume when we rotate the area between $y = x^3$, $y = x$ from $x = 0$ to $x = 1$ about $y = 3$

$$v = \pi \int_0^1 (3-x)^2 - (3-x^3)^2 \, dx$$



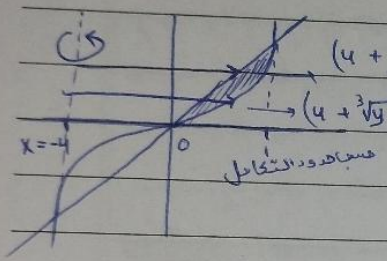
Subject: Volume Tuesday 12 / 12 / 2023

Example: Find the volume when we rotate the area between $y = \sin x$, $y = \cos(x)$ from $x = 0$ to $x = \frac{\pi}{4}$ about $y = 5$



$$V = \pi \int_0^{\frac{\pi}{4}} (5-\sin(x)^2) - (5-\cos(x))^2 \, dx$$

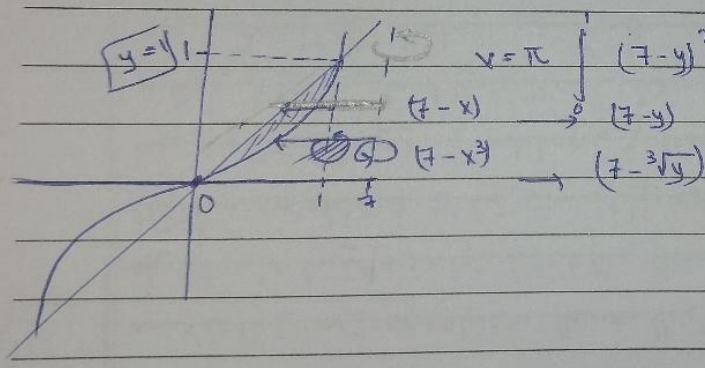
Example: Find the V $y = x$, $y = x^3$ from $x = 0$ to $x = 1$ about $x = -4$



$$V = \pi \int_{y=0}^{y=1} (4+\sqrt[3]{y})^2 - (4+y)^2 \, dy$$

حدود التكامل بدلالة y

Example: Find the V $y = x^3$, $y = x$ from $x = 0$ to $x = 1$ about $x = 7$



$$V = \pi \int_0^1 (7-y)^2 - (7-\sqrt[3]{y})^2 \, dy$$:

* هتاخدي حدود التكامل بدلالة y

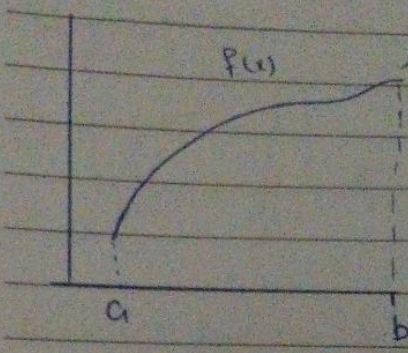
بتروحي الى اللي بدلالة x في اي اقتران

$1 \rightarrow y = x^3 \rightarrow 1^3 = 1$

$x - 2 = 8y$

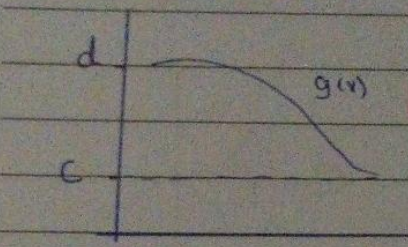
Subject: Arc Length طول القوس Tuesday 12/12/2023

معطيات السؤال: f(x) والحدود



arc length

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$



$$L = \int_c^d \sqrt{1 + (g'(x))^2}\, dx$$

Example: Find the length ~~from~~ $y = x^{\frac{3}{2}}$ from $x=1$ to $x=4$

* solution: $L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

* $f'(x) = \frac{3}{2} x^{\frac{1}{2}} \rightarrow \frac{3}{2}\sqrt{x}$

$(f'(x))^2 = \frac{9}{4} x$

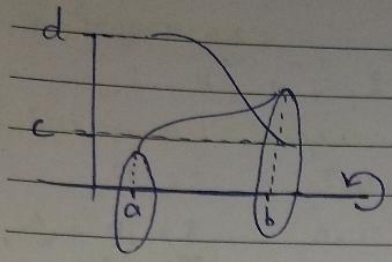
$$L = \int_1^4 \sqrt{1 + \left(\frac{3}{2}\sqrt{x}\right)^2}\, dx = \int_1^4 \sqrt{1 + \frac{9}{4}x}\, dx$$

table $= \int_1^4 \frac{\left(1 + \frac{9}{4}x\right)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)\frac{9}{4}}$

* set up the integral تجهيز التكامل ← ماتكامل فقط نجهز

مساحة سطح الدوران

Subject: Area of surface of revolution Sunday 17 / 12 /2023



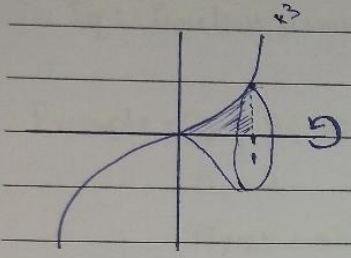
about x axis :-

$$S = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2}\, dx$$

about y axis :-

$$S = \int_c^d 2\pi g(y) \sqrt{1 + (g'(y))^2}\, dy$$

Example : find the Area of surface of revolution when we rotate $y = x^3$ from $x=0$ to $x=1$ about x-axis :- كل منحنى الدوران حول المحور السيني



solution: $S = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2}\, dx$

$$S = \int_0^1 2\pi x^3 \sqrt{1 + (3x^2)^2}\, dx$$

$$2\pi \int_0^1 x^3 \sqrt{1 + 9x^4}\, dx = \int_1^{10} \frac{2\pi}{36} \sqrt{u}\, du$$ * توزيعات

Example: set up the integral of S of $y = \cos(x)$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{2}$

about x axis :-

$$S = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2}\, dx \rightarrow \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\pi \cos(x) \sqrt{1 + (-\sin x)^2}$$

Subject: Sequences and series Sunday 17 / 12 / 2023

* Sequences:

2, 4, 6, 8, 10, 12 ← قيمة الحد

n=1 n=2 n=3 n=4 n=5 n=6

first term $n=1$ رقم الحد

$f(1)=2$ $f(2)=4$ $f(3)=6$ $f(4)=8$

تعويض $a_1=2$ $a_2=4$ $a_3=6$ $a_4=8$

$\{a_n\}_{n=1}^{\infty}$ → sequence :-

$= \{a_1, a_2, a_3, a_4, \dots\}$

$= \{2, 4, 6, 8, \dots\}$

Sq: function domain natural number and range real number.

Example: ① $\{5, 10, 15, 20, \dots\}$

$a_1=5$ $a_2=10$ $a_3=15$ $a_4=20$ $a_6=30$

a_{271} → General term: $a_n = 5n$

$271 \times 5 = 1355$

Example: find the general term :-

① $\{6, 12, 18, 24, 30, \dots\}$ → $a_n = \{6n\}_{n=1}^{\infty}$

② $\{0, 0, 0, 0, 0, \dots\}$ $a_n = \{0n\}$ constant sequence

③ $\{\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots\}$ $a_n = \{\frac{n+1}{n+2}\}$

④ $\{-1, 1, -1, 1, 1, -1, \dots\}$ $a_n = \begin{cases} n \text{ odd} \rightarrow -1 \\ n \text{ even} \rightarrow 1 \end{cases}$ او $\{-1^n\}_{n=1}^{\infty}$

⑤ $\{2, 4, 8, 16, 32, 64, \dots\}$ $a_n = \{2^n\}$

⑥ $\{8, 13, 18, 23, \dots\}$ $a_n = \{5n+3\}$

⑦ $\{1, 4, 9, 16, \dots\}$ $a_n = \{n^2\}$

⑧ $\{12, \_, 23, \_, 34, \_, 45, \_, 56, \dots\}$ $a_n = \{11n+1\}$

← continue

Subject: / /

9) $\{10, 21, 32, 43, 54, \dots\}$ $a_n = \{11n - 1\}$

10) $\{1, -1, 1, -1, 1, -1, \dots\}$ ① $a_n = \begin{cases} 1 & n \text{ odd} \\ -1 & n \text{ even} \end{cases}$

② $a_n = -1(-1)^n \rightarrow (-1)^{n+1}$

*Example:

Let $a_n = 4n+1$, find the first three terms of a_n:

S: $a_1 \rightarrow 4(1)+1 = 5$

$a_2 \rightarrow 4(2)+1 = 9$

$a_3 \rightarrow 4(3)+1 = 13$ // $a_{30} = 4(30)+1 = 121$

Example:

Let $a_1 = 4$, $a_{n+1} = \frac{n}{a_n}$ find a_2, a_3, a_4

عوض 1 طلع وسبها 2 وهذا السؤال

a_2 المطلوب

1) $a_2 = a_{1+1} = \frac{1}{a_1} = \frac{1}{4}\, a_2$

2) $a_3 = a_{2+1} = \frac{2}{\frac{1}{4}} = 8$ بدلالة اللي قبل

3) $a_4 = a_{3+1} = \frac{3}{a_3} = \frac{3}{8}$

Example:

$a_1 = 3$ $a_2 = 4$ $a_n = a_{n-2} + (a_{n-1})^2$

Find a_3 =

عوض مكان n بـ 3

$a_3 = a_{3-2} + (a_{3-1})^2 \rightarrow a_3 = 3 + (4)^2 \rightarrow 19$

$a_4 = a_2 + (a_3)^2 \rightarrow a_4 = 4 + (19)^2 = 365$

Example: sketch

$a_n = \{2, 4, 6, 8, \dots\}$ ① المدى قيم

$n=1$ $n=2$ $n=3$ $n=4$ $\rightarrow 2n$

طبعاً بوصل النقاط لأنها في اعداد حقيقية