Subject:

* مسائل مهمة لازم أميز بينهم .

* تربيع أو تكعيب أو جذر نفس الحل :

$\ln \xRightarrow{\text{نفسه}} \log_e$

① $\int \ln x^2 \, dx \Rightarrow$ بقدر أطلع 2 برا

$\int 2 \ln x \, dx \Rightarrow 2\,(x \ln x - x) + c$

② $\int \underbrace{\log_7 x}_{u} \; \underbrace{dx}_{dv}$

$u = \log_7 x \Rightarrow du \quad \frac{\ln 7}{x}$

$dv = 1dx \Rightarrow v = x \, dv$

$\int \log_7 x \, dx = x \log_7 x - \int \frac{\ln 7}{\cancel{x}} \, dx$

$x \log_7 x - \ln 7 \, x + c$

$\log_a u \rightarrow \frac{u'}{u \ln(a)}$

Subject: Integration by parts 29 / 10 / 2023

Example:

1) $\int \underbrace{\tan^{-1}x}_{u}\ \underbrace{dx}_{dv}$ ⇒ by parts → $du = \frac{1}{1+x^2}dx$ $\frac{1}{1+x^2}$ ⇐ مشتقة $\tan^{-1}$

$\int \tan^{-1}x\,dx \Rightarrow x\tan^{-1}x - \int \frac{x}{1+x^2}$ $v = x$ ⇒ dx

تعويض

$w = 1+x^2$

$\int \tan^{-1}x\,dx = x\tan^{-1}x - \int \frac{\cancel{x}}{w}\frac{dw}{2\cancel{x}}$ $dw = 2x\,dx \Rightarrow dx\,\frac{dw}{2x}$

$x\tan^{-1}x - \frac{1}{2}\int\frac{dw}{w} \Rightarrow x\tan^{-1}x \frac{-1}{2}\ln|1+x^2| + c$

Do

② $\int \underbrace{\sin^{-1}x}_{u}\ \underbrace{dx}_{dv}$ $du\ \frac{1}{\sqrt{1-x^2}}$ مشتقة $\sin^{-1}$ هو

$\int \sin^{-1}x\,dx = x\sin^{-1}x - \int x \cdot \frac{1}{\sqrt{1-x^2}}$ ← subtitution $u = \sin^{-1}x$

$du = dx$, $v = x$

$\int x \cdot \frac{1}{\sqrt{c}} \frac{dc}{-2x}$ التعويض

$c = 1-x^2 \rightarrow dc = -2x\,dx$

$\int \sin^{-1}x\,dx = x\sin^{-1}x + \frac{1}{2}\int\frac{1}{\sqrt{c}} \rightarrow \int c^{-\frac{1}{2}} = \frac{c^{\frac{1}{2}}}{\frac{1}{2}}$ $dx = \frac{dc}{-2x}$

③ $\int \underbrace{\tan^{-1}(4x)}_{u}\ \underbrace{dx}_{dv}$ $x\sin^{-1}x + \frac{1}{2}(2\sqrt{1-x^2}) + c$

$\frac{1}{1+x^2}$ = مشتقة $\tan^{-1}$

$\int \tan^{-1}(4x)\,dx \rightarrow x\tan^{-1}(4x) - \int x \cdot \frac{1}{1+(4x)^2 \rightarrow 16x^2}$ $u = \tan^{-1}(4x)$

$x\tan^{-1}(4x) - \int x\,\frac{1}{z}\,\frac{dz}{32x}$ $du = \frac{1}{1+(4x)^2}$

$x\tan^{-1}(4x) - \frac{1}{32}\int\frac{dz}{z} \rightarrow \ln(z)$ $dv = dx$

$v = x$

$x\tan^{-1}(4x) \frac{-1}{32}\ln(1+(4x)^4) + c$ subtitution

$z = 1+16x^2$

$dz = 32x\,dx$

$dx = \frac{dz}{32x}$

← continue

Subject: ______ / /

Example:

$$1)\ \int x^3 e^{x^2}\,dx \Rightarrow \int x^3 e^u \frac{du}{2x} \Rightarrow \frac{1}{2}\int u e^u\,du$$

هون بجيب أجزاء

sub: $u = x^2 \Rightarrow du = 2x\,dx$

$dx = \frac{du}{2x}$

$$= ue^u - \int e^u\,du$$

$$u e^u - e^u$$

$$x^2 e^{x^2} - e^{x^2} + c$$

$u = u \Rightarrow du = 1$

$dv = e^u \Rightarrow v = e^u$

$$② \int \sin(\ln x)\,dx$$

$$\int \sin(\ln x)\,dx = \sin(\ln x)\cdot x - \int x\,\cancel{dx}\,\frac{\cos(\ln x)}{x}$$

كمان أجزاء

$u = \sin(\ln x) \Rightarrow du = \cos(\ln x)\cdot\frac{1}{x}$

$dv = dx \Rightarrow v = x$

$$x\sin(\ln x) - \int \underbrace{\cos(\ln x)}_{w}\,\frac{dx}{dz}$$

$w = \cos(\ln x)\quad dw = \frac{-\sin(\ln x)}{x}$

$z = x\quad dz = dx$

$$\int \sin(\ln x)\,dx = x\sin(\ln x) - (x\cos(\ln x)) + \int \frac{+\sin(\ln x)}{\cancel{x}}\cdot\cancel{x}$$

$$\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)$$

[illegible]

$$\frac{2}{2}\int \sin(\ln x)\,dx = \frac{x\sin(\ln x) - x\cos \ln x)}{2}$$

$$\int \sin(\ln x)\,dx = \frac{x\sin(\ln x) - x\cos(\ln x)}{2}$$

Do

$$③ \int \cos(\ln x)\,dx$$

$$\int \cos(\ln x)\,dx = x\cos(\ln x) + \int \frac{x\sin(\ln x)}{x}$$

أجزاء

$u = \cos(\ln x)$

$du = \frac{-\sin(\ln x)}{x}$

$dv = dx \Rightarrow v = x$

$$x\cos(\ln x) + (x\sin(\ln x) - \int \cos(\ln x)$$

$\int \sin(\ln x)\,dx$:

$u = \sin(\ln x)$

$du = \frac{\cos(\ln x)}{x}$

$dv = dx$

$v = x$

$$\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)$$

$$+\int \cos(\ln x) \qquad + \int \cos(\ln x)$$

$$\int \cos(\ln x) = \frac{1}{2}(x\cos(\ln x) + x\sin(\ln x) + c$$

Subject: 29 / october 2023

Example

1) $\int e^x \cos x \, dx = e^x \sin x - \int \underbrace{e^x \sin x \, dx}_{\text{by parts}}$

$u = e^x \quad du = e^x$
$dv = \cos x \, dx$
$v = \sin x$

$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x$

$e^x \to e^x$
$\sin x \to -\cos x$

$u = e^x \quad du = e^x$
$v = -\cos x \quad dv = \sin x$

$e^x \sin x - \left(-e^x \cos x - \int -\cos x \cdot e^x\right.$

$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int \cos x \cdot e^x \, dx$

$+\int e^x \cos x \, dx = \qquad + \int \cos x \cdot e^x \, dx$

$= \dfrac{e^x \sin x + e^x \cos x}{2} + C$

غلط

2) $\int x^2 \ln x \, dx \Rightarrow \int x^2 \, u \; x \, du$

$= \int x^3 \Rightarrow$ غلط

$u = \ln x$
$du = \frac{1}{x} dx$
$dx = x \, du$

$\int x^2 \ln x \, dx \Rightarrow \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx$

$u = \ln x \Rightarrow du \; \frac{1}{x} dx$
$dv = x^2 \, dx \Rightarrow \frac{x^3}{3}$

$\ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2$

كثيرحدود مع لوغاريتم

$\frac{x^3}{3} \ln x - \frac{1}{9} x^3 + C$

3) $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2) \, d\theta \Rightarrow \int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta^3 \cos u \, \frac{du}{2\theta} \quad \frac{1}{2} \int u \cos u \, du$

تعويض :-

$u = \theta^2 \quad du = 2\theta \, d\theta$
$d\theta = \frac{du}{2\theta}$

$\int \theta^3 \cos \theta^2 \, d\theta \rightarrow \frac{1}{2}\left(u \sin u - \int \sin u\right)$

by parts

$= \left.\frac{1}{2} \theta^2 \sin \theta^2 + \cos \theta^2 \right|_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} = \frac{1}{2}\left(\pi \sin \pi + \cos \pi\right) - \left(\frac{\pi}{2} \sin \frac{\pi}{2} + \cos \frac{\pi}{2}\right)$

0 1 1

$u = u \quad du = 1$
$v = \sin u \quad dv = \cos u$

$\frac{1}{2}(-1) - \left(\frac{\pi}{2} \times 1 + 0\right)$

4) $\int \cancel{x^2 \ln x \, dx}$ $\quad \int \ln \sqrt{x} \, (x^3 + 3) \, dx$

Subject: Section 7.3 29 / oct / 2023

Section 7.3: Trigonometric Integral

I) $\int \sin^n x\,dx$, $\int \cos^n x\,dx$, $\int \sin^n x \cdot \cos^m x\,dx$ sin, cos نفس طريقة الحل

II) $\int \tan^n x\,dx$, $\int \cot^n x\,dx$

III) $\int \sec^n x\,dx$, $\int \csc^n x\,dx$

IV) $\int \tan^n x \sec^m dx$, $\int \cot^n x \csc^m x\,dx$

* Power of sin x :

1) $\int \sin^2(x)\,dx \Rightarrow \int \frac{1}{2}(1-\cos(2x))\,dx \Rightarrow \frac{1}{2}\left(x - \frac{\sin(2x)}{2}\right) + C$

2) $\int \sin^3(x)\,dx \Rightarrow \int \sin^2(x)\sin(x)\,dx$ نعوض متطابقة و فردي بنفصل

$\int (1-\cos^2(x))\sin(x)\,dx$

$\int (\sin(x)\,dx - \int \cos^2(x)\sin x\,dx$ تعويض

$-\cos(x) - \int u^2 \sin x \frac{du}{-\sin x}$ $u = \cos(x)$

$du = -\sin(x)\,dx$

$-\cos(x) + \frac{(\cos(x))^3}{3} + C$ $dx = \frac{du}{-\sin x}$

* $\int \sin^n x\,dx = \frac{-1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2} x\,dx$ $n \geq 2$

$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$

Example:

$\int \sin^4 x\,dx = \frac{-1}{4}\sin^3(x)\cos(x) + \frac{3}{4}\int \sin^2(x)\,dx$

$= \frac{-1}{4}\sin^3(x)\cos(x) + \frac{3}{4}\left(\frac{1}{2}\frac{(x-\sin 2x)}{2}\right) + C$

Subject: ________ Tuesday 30 / 10 / 2023

* Power of tan(x) . cot(x) :-

1) $\int \tan x \, dx \Rightarrow \int \frac{\sin x}{\cos x} dx = -\ln|\cos(x)| + c$

$\ln|\sec(x)| + c$

2) $\int \tan^2(x) \, dx \Rightarrow \int (\sec^2(x) - 1) \, dx = \tan(x) - x + c$ $1 + \tan^2 = \sec^2$

3) $\int \tan^3(x) \, dx \Rightarrow \int \tan^2(x) \tan(x) \, dx = \int \tan(x) (\sec^2(x) - 1) \, dx$ $\int \cot = \ln \sin$

$\rightarrow \int \underbrace{\tan(x)}_{\text{تعويض}} \sec^2(x) \, dx - \int \underbrace{\tan(x)}_{\ln|\sec(x)|} dx$

4) $\int \tan^4(x) \, dx \Rightarrow \int \tan^2(x) \tan^2(x) \, dx$

$\int (\underbrace{\sec^2(x)}_{u = \tan} - 1) \tan^2(x) \rightarrow \int \sec^2(x) \tan^2(x) - \int \tan^2(x) \, dx$

5) $\int \tan^5(x) \, dx \Rightarrow \int \tan^2(x) \tan^3(x) \, dx \Rightarrow \int (\sec^2(x) - 1) \tan^3(x) \, dx$

* Power of sec(x) . csc(x) :-

1) $\int \sec x \, dx \Rightarrow \int \sec(x) \cdot \frac{\tan(x) + \sec(x)}{\tan(x) + \sec(x)} dx$

$= \int \frac{\sec(x)\tan(x) + \sec^2(x)}{\tan(x) + \sec(x)} dx \Rightarrow \ln|\tan(x) + \sec(x)| + c$

2) $\int \sec^3(x) \, dx = \int \underbrace{\sec(x)}_{u} \cdot \underbrace{\sec^2(x)}_{dv} dx$ ما بزبط تعويض وتوزيع على المتطابقة !

by parts

$\int \sec^3(x) dx = \sec(x)\tan(x) - \int \tan(x) \sec(x) \tan(x) \, dx$ $u = \sec(x)$ $du = \sec(x)\tan(x)$

$dv = \sec^2(x)$ $v = \tan(x)$

$\int \sec^3(x) dx = \sec(x) \tan(x) - \int \tan^2(x) \sec(x) \, dx$

$\int \sec^3(x) dx = \sec(x) \tan(x) - \int (\sec^2(x) - 1) \sec(x) \, dx$

$\int \sec^3(x) dx = \sec(x) \tan(x) - \int \sec^3(x) \, dx + \int \sec(x) \, dx$

$2 \int \sec^3(x) dx = \sec(x) \tan(x) + \ln|\tan(x) + \sec(x)| + c$

⇐ continue

Subject: Tuesday 30 / october / 2023

Power of $\sin x$ and $\cos x$:-

$\int \sin^n(x)\ \cos^m(x)\,dx$ — لازم نفس الزاوية .

1) $\int \sin^n(x) \cos^m(x) \rightarrow n = \text{even} \ , \ m = \text{odd} \rightarrow u = \sin x$

Example:

$\int \sin^3(x)\cos^2(x)\,dx = \rightarrow -\int \sin^3(x)\ u^2 \cdot \frac{+du}{\sin(x)}$ — $u = \cos(x)$

$\int \sin^3(x)\cos^2(x)\,dx = -\int \sin^2(x)\ u^2\ du$ — $du = -\sin(x)\,dx$

$= -\int (1-u^2)\,u^2\,du \Rightarrow -\int u^2 - u^4\ du$ — $dx = \frac{-du}{\sin(x)}$

$-\left(\frac{\cos^3(x)}{3} - \frac{\cos^5(x)}{5}\right) + C$

Do: $\int \sin^7(11x)\ \cos^4(11x)\,dx$

~~$\int \sin^7(11x) \cos t$~~

2) $\int \sin^n(x)\cos^m(x)\,dx \Rightarrow n, m \rightarrow \text{odd} \rightarrow u = \text{greatest power}$

Example:

$\int \sin^7(x) \cdot \cos^3(x)\,dx \rightarrow \int u^7 \cdot \cos^3(x)\ \frac{du}{\cos x}$ — $u = \sin x$

$du = \cos x\,dx$

$\int \sin^7(x)\cdot\cos^3(x)\,dx \rightarrow \int u^7 \cdot (1-u^2)\,du \rightarrow \int u^7 - u^9\ du$ — $dx = \frac{du}{\cos x}$

$\frac{\sin^8 x}{8} - \frac{\sin^{10} x}{10} + C$

3) $\int \sin^n(x)\cos^m(x)\,dx \rightarrow n, m \text{ even} \rightarrow u =$ — سؤال final

Example:

$\int \sin^4 x \cdot \cos^4 x\ dx \rightarrow \int \left(\frac{1}{2}(1-\cos(2x))\right)^2 \cdot \left(\frac{1}{2}(1+\cos(2x))\right)^2 dx$ — مطابقة $\cos^2, \sin^2$

$\int \sin^4 x \cdot \cos^4 x\,dx \rightarrow \frac{1}{16}\int (1-\cos(2x))^2 \cdot (1+\cos(2x))^2$ — فرق بين مربعين

$\rightarrow \frac{1}{16}\int (1-\cos(2x)) \cdot (1+\cos(2x))^2$

$\frac{1}{16}\int (1-\cos^2(2x))^2\,dx \rightarrow \frac{1}{16}\int (\sin^2 2x)^2\,dx$

$\frac{1}{16}\int (\sin 2x)^4$

← continue

Subject: Power of functions 30/octobe / 2023

$\int \tan^n x \ \sec^m x \ dx$

example:

1) $\int \tan^6 x \ \sec^4 x \ dx \rightarrow \int u^6 \sec^4 x \frac{du}{\sec^2(x)} \rightarrow \int u^6 (1+u^2)\, du$ تعويض sec و tan

$u = \tan x$

$\int u^6 + u^8 \, du$ $du = \sec^2 x \, dx$

2) $\int \tan^3 x \ \sec^5 x \, dx \rightarrow \int \tan^3 x \ u^5 \frac{du}{\sec(x)\tan(x)}$ اذا ال Tan فردي بالعكس $u = \sec x$

$= \int (u^2-1)\, u^4 \, du$ $u = \sec x$

$\int u^6 - u^4 \, du$

3) $\int \tan^2 x \ \sec x \, dx = \int (\sec^2(x) - 1) \sec(x) dx$

اجزاء $\leftarrow \int \underline{\sec^3(x)} - \sec(x) dx$

* Idenlities:

- $\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$ الزوايا خليه
- $\cos A . \cos B = \frac{1}{2}((\cos(A-B) + \cos(A+B))$
- $\sin A . \cos B = \frac{1}{2}((\sin(A-B) + \sin(A+B))$

Example:

1) $\int \sin(10x) . \cos(6x) dx \rightarrow \frac{1}{2}\int \sin(4x) + \sin(16x)\, dx$

$\frac{1}{2}\left(\frac{-\cos 4x}{4} + \frac{-\cos(16x)}{16} \right) + c$

2) $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} dx \rightarrow \int \frac{\sin^3 u}{\cancel{\sqrt{x}}} du\, 2\cancel{\sqrt{x}} \rightarrow 2\int \sin^3 u \, du$ $u = \sqrt{x}$

$-2\cos\sqrt{x} + \frac{2}{3}\cos\sqrt{x} + c$ * ما بتختصر زاوية $du = \frac{1}{2\sqrt{x}} dx$

* الزاوية ما بجيب العب بيها $dx = 2\sqrt{x}\, du$

3) $\int \frac{x}{u} \frac{\sin^3 x}{dv} dx \longrightarrow$ by parts

4) $\int \sqrt{\tan(x)} \ \sec^4(x) \ dx \rightarrow \int \sqrt{u} \ \sec^4(x) \frac{du}{\sec^2(x)} \rightarrow \int \sqrt{u} \ \sec^2(x) du$ $u = \tan(x)$

$\int \sqrt{u} \ (\tan^2(x) + 1)\, du \rightarrow \int \sqrt{u}\ (u^2+1)\, du$ $du = \sec^2(x)\, dx$

$\int u^{\frac{5}{2}} + u^{\frac{1}{2}} \, du \rightarrow$

تعويضات مثلثية

Subject: 7.4 trigonometric substitution   Sunday   5 / November 2023

① $\sqrt{x^2+a^2}$ $x = a\tan\theta$

② $\sqrt{a^2-x^2}$ $x = a\sin\theta$

③ $\sqrt{x^2-a^2}$ $x = a\sec\theta$

* Example :

1) $\int \frac{1}{\sqrt{x^2-9}}\,dx \Rightarrow a=3 \Rightarrow x = 3\sec\theta$ راح أخلي السؤال بدلالة θ

$\int \sqrt{x^2-9} \rightarrow \sqrt{(3\sec\theta)^2 - 9}$ $dx = 3\sec\theta\tan\theta\, d\theta$

توزيع التربيع $= \int \sqrt{(9\sec^2\theta) - 9} \rightarrow \int 3\sqrt{\sec^2\theta - 1} \rightarrow \int 3\tan\theta \rightarrow$ هون أطلعها من الجذر

$\int \frac{1}{3\tan\theta} \Rightarrow \int \sec\theta\, d\theta \rightarrow \ln|\tan\theta + \sec\theta| + c$ أرجع لدلالة x

$\ln\left|\tan\left(\sec^{-1}\frac{x}{3}\right) + \frac{x}{3}\right| + c$ $\sec^{-1}\frac{x}{3} = \theta \leftarrow \frac{x}{3\sec} = \frac{3\sec\theta}{3\sec}$

2) $\int \frac{x^2}{\sqrt{3-x^2}}\,dx \Rightarrow a=\sqrt{3} \Rightarrow x = \sqrt{3}\sin\theta$ $dx = \sqrt{3}\cos\theta\, d\theta$

$\sqrt{3-x^2} = \sqrt{3-(\sqrt{3}\sin\theta)^2} \rightarrow \sqrt{3-3\sin^2\theta}$

$\sqrt{3(1-\sin^2)} \rightarrow \sqrt{3\cos^2\theta} \rightarrow \sqrt{3}\cos\theta \rightarrow$ قيمة الجذر اللي في المقام

$\int \frac{3\sin^2\theta}{\sqrt{3}\cos\theta}\,d\theta \rightarrow \int \frac{3\sin^2\theta}{\sqrt{3}\cancel{\cos\theta}}\sqrt{3}\cancel{\cos\theta} = 3\int \sin^2\theta\, d\theta$

$3\int \frac{1}{2}(1-\cos 2\theta)\,d\theta \rightarrow \frac{3}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)d\theta \Rightarrow \frac{x}{\sqrt{3}} = \sin\theta \rightarrow \theta = \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$

$\frac{3}{2}\left(\sin^{-1}\frac{x}{\sqrt{3}} - \frac{\sin 2\left(\sin^{-1}\frac{x}{\sqrt{3}}\right)}{2}\right) + c$

3) $\int \frac{1}{x^2\sqrt{x^2+16}}\,dx \Rightarrow x = 4\tan\theta$ $dx = 4\sec^2\theta\, d\theta$

$\sqrt{x^2+16} \rightarrow 4\sec\theta$

4) $\int \frac{dx}{x^2\sqrt{4x^2-9}} \rightarrow \int \frac{dx}{x^2 \cdot 2\sqrt{x^2-\frac{9}{4}}} \rightarrow a = \frac{3}{2},\ x = \frac{3}{2}\sec\theta$

5) $\int e^x\sqrt{1-e^{2x}}\,dx \rightarrow \int e^x\sqrt{1-e^{x^2}}\,dx \rightarrow \int \cancel{u}\sqrt{1-u^2}\,\frac{du}{\cancel{e^x}}$ $e^{2x} = (e^x)^2$

تكملة $\int\sqrt{1-u^2}\,du$ $u = e^x \rightarrow dx = \frac{du}{e^x}$

6) $\int \frac{dx}{\sqrt{1-(x-1)^2}} \rightarrow (x-1) = (\overset{a}{1}\cdot\sin\theta)$ أو نفرضها y $(x-1)\sin^{-1} + c \rightarrow ?$

7) $\int \frac{dx}{\sqrt{16+(x+7)^2}}$

Subject: Recall

Tuesday 7 / ~~october~~ November / 2023

Ex: ① $\int \frac{dx}{(4x^2-9)^{\frac{3}{2}}} \rightarrow \int \frac{dx}{\sqrt{(4x^2-9)^3}}$

*أسئلة مهمة

الحل: $\int \frac{dx}{\sqrt{(4(x^2-\frac{9}{4}))^3}} \rightarrow \int \frac{dx}{\sqrt{4^3}\sqrt{(x^2-\frac{9}{4})^3}} \rightarrow x = \frac{3}{2}\sec\theta$

② $\int \frac{\cos(x)}{\sqrt{2-\sin^2 x}}\,dx \rightarrow \int \frac{\cos(x)}{\sqrt{2-u^2}} \frac{du}{\cos(x)} \rightarrow \int \frac{du}{\sqrt{2-u^2}} \rightarrow u = \sqrt{2}\sin\theta$ $\begin{cases} u = \sin x \\ du = \cos x\, dx \end{cases}$

$\sqrt{2-u^2} = \sqrt{2}\cos\theta$

$\int \frac{\sqrt{2}\cos\theta}{\sqrt{2}\cos\theta}\,d\theta \rightarrow \int d\theta \rightarrow \theta + c \rightarrow \sin^{-1}\left(\frac{u}{\sqrt{2}}\right) + c$ $du = \sqrt{2}\cos\theta\, d\theta$

رجعها بدلالة x

③ $\int \frac{dx}{\sqrt{x^2-6x+10}} \xrightarrow{\text{اكمال مربع}} x^2-6x+9-9+10 \rightarrow (x-3)^2+1$ لازم لإكمال المربع معامل $x^2 = 1$

$\int \frac{dx}{(x-3)^2+1} \rightarrow \sqrt{(x-3)^2+1} = \sqrt{\tan^2\theta+1} \rightarrow \sec\theta$

④ $\int \frac{dx}{\sqrt{6x-2x^2+10}} \rightarrow \int \frac{dx}{\sqrt{-2(-3x+x^2-5)}}$

لحد هون كويز يوم الثلاثاء !

* Section 7.5

Integration of rational functions: $\frac{\text{Poly}}{\text{Poly}}$ قوة x عدد طبيعي ← (مش سالب ولا جذر) كثير حدود

I) $\int \frac{P(x)}{Q(x)}\,dx \rightarrow$ degree $(P(x)) \geq$ degree of $(Q(x)) \rightarrow$ long division $Q(x)\overline{\big)\,P(x)}$ with quotient $S(x)$

اول خطوة نقسم الـ x على بعض $\int S(x)\,dx + \int \frac{R(x)}{Q(x)}\,dx$

1) $\int \frac{x^2}{x+9}\,dx \rightarrow$

$$\begin{array}{r} x - 9 \rightarrow \text{result} \\ x+9 \overline{\big)\, x^2 \qquad} \\ -(x^2 + 9x) \\ \hline -9x \qquad \\ -(-9x - 81) \\ \hline 81 \rightarrow \text{reminder} \end{array}$$

ضرب

(درجة الباقي أقل من المقسوم عليه) بوقف قسمة

$\rightarrow \int x-9 + \int \frac{81}{x+9}\,dx$

$\frac{x^2}{2} - 9x + 81\ln|x+9| + c$

2) $\int \frac{x^3+x}{x+1}\,dx \rightarrow$

$$\begin{array}{r} x^2 - x + 2 \\ x+1 \overline{\big)\, x^3 + x} \\ -(x^3 + x^2) \\ \hline -x^2 + x \\ -(-x^2 - x) \\ \hline 2x \\ -(2x + 2) \\ \hline -2 \end{array}$$

ما في x من درجتها

$\rightarrow \int x^2-x+2 + \int \frac{-2}{x+1}$

$\frac{x^3}{3} - \frac{x^2}{2} + 2x - 2\ln|x+1| + c$