Subject: Recall   Sunday   12 / 11 / 2023

Rational functions:

Example:

1) $\int \frac{x^3+1}{x^2+1}\,dx \Rightarrow x^2+1 \overline{)\,x^3+1}$ quotient x

$-x^3 \mp x$

$-x+1$

$\Rightarrow \int x\,dx + \int \frac{-x+1}{x^2+1}\,dx$

$\frac{x^2}{2} + \frac{1}{2}\int \frac{-2x}{x^2+1} + \int \frac{1}{x^2+1}$ توزيع المقام

ضرب ب 2   أو تعويض

$\frac{x^2}{2} - \frac{1}{2}\int \frac{2x}{x^2+1} + \int \frac{1}{x^2+1}$

$\frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| + \tan^{-1}(x) + c$

2) $\int_1^2 \frac{x}{x+9}\,dx \Rightarrow x+9 \overline{)\,x}$ quotient 1

$-x \mp 9$

-9

$\Rightarrow \int 1\,dx + \int \frac{-9}{x+9}\,dx$

$= x - 9\ln|x+9| \Big|_1^2$

$(2-9(\ln|2+9|) - (1-9(\ln|1+9|)$

$2-9\ln|11| - 1 + 9\ln|10| \neq 1$ $\boxed{\log_{10} 10 = 1}$

* $\int \frac{P(x)}{q(x)}\,dx \rightarrow$ degree $p(x) <$ degree $q(x)$

Partial fraction   كسور جزئية

Ex: $\int \frac{1}{x^2-49}\,dx \Rightarrow \int \frac{1}{(x+7)(x-7)}\,dx$

ضرب بالمقام (الاختزال الكامل)

عدد الثوابت حسب درجة المقام

$\frac{1}{x^2-49}\times(x^2-49) = \frac{A}{(x-7)}(x^2-49) + \frac{B}{(x+7)}(x^2-49) \rightarrow 1 = A(x+7) + B(x-7)$

افرض أي قيمة ل x و لكن الأفضل تصير القوس صفر

$1 = 14A \rightarrow A = \frac{1}{14} \leftarrow x = 7$

$1 = -14B \rightarrow B = \frac{-1}{14}$ $x = -7$

$\int \frac{1}{x^2-49}\,dx = \int \frac{\frac{1}{14}}{(x-7)} + \int \frac{\frac{-1}{14}}{(x+7)}\,dx \Rightarrow \frac{1}{14}\ln|x-7| - \frac{1}{14}\ln|x+7| + c$

تحليل ثم توزيع ثم ضرب بالمقام

2) $\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx \xrightarrow[\text{fraction}]{\text{Partial}}$ (ضرب بالمقام) $= \left(\frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}\right)$

قيم $x = 1, -1, -3$:

$x^2+4x+1 = (A(x+1)(x+3)) + (B(x-1)(x+3)) + (C(x+1)(x-1))$

$x=1 \rightarrow 1+4+1 = A(2\cdot4) + \text{zero } B + \text{zero } C \rightarrow A = \frac{6}{8} \Rightarrow \frac{3}{4}$

$x=-1 \rightarrow 1-4+1 = \text{zero } A + B(-2\cdot2) + \text{zero } C \rightarrow B = \frac{1}{2}$

$x=-1 \rightarrow 1-4+1 = \text{zero } A + \text{zero } B + C(-2\cdot-4) \rightarrow C = \frac{-1}{4}$

$x=3 \rightarrow -3^2 - 12 + 1 = \text{zero } A + \text{zero } B + C$

Example 2: تكملة

$$\int \frac{x^2+4x-1}{(x+1)(x-1)(x+3)} = \int \frac{\frac{3}{4}}{(x-1)}\,dx + \int \frac{\frac{1}{2}}{(x+1)}\,dx + \int \frac{\frac{-1}{4}}{(x+3)}\,dx$$

$$\frac{3}{4}\ln|x-1| + \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln|x+3| + C$$

← continue

Subject: rational function   Sunday   12/November/2023

3) $\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx \rightarrow \int \frac{x^2+2x-1}{x(2x^2+3x-2)}\,dx \rightarrow x(2x-1)(x+2)$

تعويض $\int \frac{x^2+2x-1}{x(2x-1)(x+2)}\,dx \rightarrow \left(\frac{A}{x}+\frac{B}{(2x-1)}+\frac{C}{(x+2)}\right)\cdot x(2x-1)(x+2)$ قيم x = صفر ، $\frac{1}{2}$ ، -1

$= A(2x-1)(x+2) + B(x)(x+1) + C(x)(2x-1)$ → continue

$\frac{1}{2}\ln|x| + 10\ln|x+2| + \frac{1}{10}\ln|2x-1| + C$ جواب مش من الدكتور

* Example:

$\int \frac{2x+4}{x^3-4x^2}\,dx \rightarrow \frac{A}{(x-4)} + \frac{B}{x} + \frac{C}{x^2}$ فكرة x^3 عدم ذوات

جوابي $-\frac{3}{4}\ln|x| + \frac{1}{x} + \frac{3}{4}\ln|x-4| + C$

* Recall:

1) $\int \frac{dx}{x^2+8x-9} \rightarrow \int \frac{1}{(x+9)(x-1)}\,dx \rightarrow \frac{A}{(x+9)} + \frac{B}{(x-1)}$ المقام $\times$ →

$\frac{1}{10}\ln|x-1| - \frac{1}{10}\ln|x+9|$

2) $\int \frac{x}{x^3-x^2-4x+4}\,dx$ أصفار العامل الأول / أصفار الحد المطلق → $\pm1, \pm4, \pm2$

قسمة طويلة أو تركيبية

عوض $x=1 \rightarrow 1-1-4+4 \Rightarrow$ zero $\rightarrow x^3-x^2-4x+4 \Rightarrow (x-1)((x-2)(x+2))$

$-\frac{1}{3}\ln|x-1| + \frac{1}{4}\ln|x-2| - \frac{1}{6}\ln|x+2| + C$

$$\begin{array}{r} x^2-4 \\ x-1\,\overline{)\,x^3-x^2-4x+4} \\ \ominus\; x^3+x^2 \\ -4x+4 \\ +4x\mp4 \\ 00 \end{array}$$

3) $\int_4^5 \frac{1}{(x-3)^2(x-7)^3}\,dx \rightarrow \frac{A}{(x-3)} + \frac{B}{(x-3)^2} + \frac{C}{(x-7)} + \frac{D}{(x-7)^2} + \frac{E}{(x-7)^3}$

ln المقام $\int (x-3)^{-2}$

4) $\int$ Find the form of partial fraction: $f(x) = \frac{1}{(x-3)^4(x+1)(x-7)^5}$

$\frac{A}{(x-3)} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)^3} + \frac{D}{(x-3)^4} + \frac{E}{(x+1)} + \frac{F}{(x-7)} + \frac{G}{(x-7)^2} + \frac{H}{(x-7)^3} + \frac{I}{(x-7)^4} + \frac{J}{(x-7)^5}$

* find the form of partial fraction: $f(x) = \frac{1}{x^2+x+1}$

$B^2-4ac<0$ المميز سالب لا تحلل

$\frac{1}{x^2+x+1} \Rightarrow \frac{Ax+B}{x^2+x+1}$

عندما لا تحلل العبارة التربيعية

Subject:

Example:

$$\int \frac{dx}{x(x^2+1)^2} \rightarrow \int \frac{A}{x} + \int \frac{Bx+C}{(x^2+1)} + \int \frac{Dx+E}{(x^2+1)^2} \, dx$$

ثوابت 4 →   (ضرب بالمقام)

$$1 = A(x^2+1)^2 + (Bx+C)\,x(x^2+1) + (Dx+E)\,x$$

$x = 0, \pm 1, \pm 2$

$A = 1$ $B = -1$ $C = 0$ $D = -1$ $E = 0$

$$\rightarrow \int \frac{1}{x} + \int \frac{-x}{(x^2+1)} + \int \frac{-x}{(x^2+1)^2} \, dx$$

① $u = x^2+1$

② $du = 2x\,dx$

الجواب   تعويض

$$\ln|x| - \frac{1}{2}\ln(x^2+1) - \qquad + C$$

Subject: Improper Integrals Sunday 19/ 11 / 2023

① $\int_a^\infty f(x)\,dx$, $\int_{-\infty}^a f(x)\,dx$, $\int_{-\infty}^{\infty} L(x)\,dx$

(case I) $\int_a^\infty f(x)\,dx = \int_a^t f(x)\,dx$ بحولها لصيغة $\rightarrow \lim_{t\to\infty} \int_a^t f(x)\,dx$

1- بكامل الاقتران integrate

2- تعويض (فوق - تحت) Subtitute

3- ناخذ النهاية take the limit

رقم ← converge متقارب

$\pm\infty$ ← diverage متباعد

Example: find the following limits if it exist:

1) $\int_2^\infty x\,dx \rightarrow$ الحل:- $\lim_{t\to\infty} \int_2^t x\,dx$

$= \lim_{t\to\infty} \frac{x^2}{2}\Big|_2^t = \lim_{t\to\infty} \left[\frac{t^2}{2} - \frac{2^2}{2}\right] = \lim_{t\to\infty} \left[\frac{t^2}{2} - 2\right]$

$= \frac{\infty^2}{2} - 2$

$\infty - 2 = \infty$

$\int_2^\infty x\,dx \rightarrow$ diveragel integral

2) $\int_1^\infty \frac{1}{x^2}\,dx \rightarrow \lim_{t\to\infty} \int_1^t \frac{1}{x^2}$

$\lim_{t\to\infty} \frac{-1}{x}\Big|_1^t \rightarrow \lim_{t\to\infty} \left(\frac{-1}{t} + 1\right) \rightarrow \lim_{t\to\infty} \left(\frac{-1}{\infty} + 1\right)$

$0 + 1 = 1$

converage

3) $\int_1^\infty \frac{1}{x}\,dx \rightarrow \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx \rightarrow \lim_{t\to\infty} \ln|x|\Big|_1^t \rightarrow \lim_{t\to\infty} (\ln t - \ln 1)$

$\ln\infty - 0$

∞ diverage

* $\int_1^\infty \frac{1}{x^p}\,dx$ → con $p > 1$ $\xrightarrow{\text{الناتج}}$ $\frac{1}{p-1}$

→ div $p \leq 1$

4) $\int_1^\infty \frac{1}{x^{10}}\,dx \rightarrow \lim_{t\to\infty} \int_1^t \frac{1}{x^{10}} \rightarrow \lim_{t\to\infty} \int_1^t x^{-10} \rightarrow \lim_{t\to\infty} \frac{x^{-9}}{-9}\Big|_1^t \rightarrow \frac{-1}{9x^9}$

$= \lim_{t\to\infty} -\left(\frac{1}{9t^9} - \frac{1}{9}\right) \rightarrow \lim_{t\to\infty} \frac{-1}{9t^9} + \frac{1}{9(1)^9}$

$-0 + \frac{1}{9} = \frac{1}{9}$

∞:

$\infty^2 = \infty$

$\infty \pm \text{number} = \infty$

$\infty + \infty = \infty$

$\frac{\infty}{\pm\text{number}} = \infty$

$\infty - \infty =$!! غير معرف

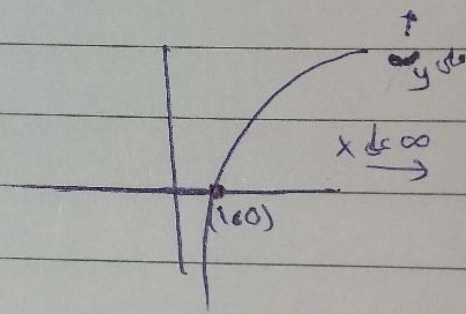
$\frac{1}{\infty} = 0$

$\frac{\text{number}}{\pm\infty} = 0$

$\frac{\infty}{\infty} =$!!

$\ln x \rightarrow$

$\ln 1 = \text{zero}$



$\ln \pm 0 =$

$\ln 0^{+} = \ln 0^{-} = -\infty$

$\ln \infty = \infty$

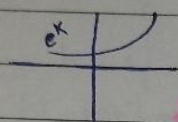
Subject: ______ / /

5) $\int_1^\infty \frac{1}{\sqrt{x}}\,dx \rightarrow \lim_{t\to\infty} \int_1^t x^{-\frac{1}{2}+\frac{2}{2}}\,dx \rightarrow \lim_{t\to\infty} \left. \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right|_1^t \rightarrow \lim_{t\to\infty} 2 \left. \sqrt{x} \right|_1^t$

$2(\sqrt{t} - \sqrt{1}) = 2(\infty - 1) = \infty \rightarrow$ diversage

6) $\int_{-\infty}^{4} e^{-3x}\,dx \rightarrow \lim_{t\to-\infty} \int_t^4 e^{-3x}\,dx \rightarrow \lim_{t\to-\infty} \left. \frac{e^{-3x}}{-3} \right|_t^4 \rightarrow \frac{-1}{3} \lim_{t\to\infty} \left. e^{-3x} \right|_t^4$

e^{∞} :-



$-\frac{1}{3} \lim_{t\to\infty} \left(e^{-3\cdot 4} - e^{-3t}\right)$

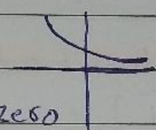
$-\frac{1}{3}\left(e^{-12} - e^{300}\right)$

$\frac{-1}{3}\left(e^{-12} - \infty\right) = \infty$ div

قاعدة

$a^{\infty} = a > 1,\ a^{\infty} = \infty$

$1 > a > 0,\ a^{\infty} = $ zero



$\lim_{t\to\infty} \left(\frac{2}{3}\right) \xrightarrow{\text{رسمة}} =$ zero

$\rightarrow \frac{2^{\infty}}{3^{\infty}} = 0$

7) $\int_0^\infty \frac{1}{x^2+1}\,dx \rightarrow \lim_{t\to\infty} \int_0^t \frac{1}{x^2+1}\,dx \rightarrow \lim_{t\to\infty} \left. \tan^{-1}(x) \right|_0^t \rightarrow \lim_{t\to\infty} \tan^{-1}(t) - \tan^{-1}(0)$

$\frac{\sin}{\cos} = \frac{1}{0}$ هي ∞ الزاوية اللي tan إلها بساوي

$\lim_{t\to\infty} \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\tan(0) = 0$

$\tan^{-1}(0) = 0$

$* \tan^{-1}(-\infty) = \frac{-\pi}{2}$

Subject: Improper Integral   Tuesday   21 / 11 / 2023

Recall:

Example:

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx \rightarrow \int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$

في مشكلتين ← بنفصل التكامل

ومنه con وحدة div ← div

$$\lim_{t\to-\infty} \int_{t}^{0} \frac{1}{1+x^2}\,dx \qquad \Big| \qquad \lim_{t\to\infty} \int_{0}^{t} \frac{1}{1+x^2}$$

$$\lim_{t\to-\infty} \tan^{-1}x \Big|_{t}^{0} \qquad \Big| \qquad \lim_{t\to\infty} \tan^{-1}(x)\Big|_{0}^{t}$$

$$\lim_{t\to-\infty} \left[\tan^{-1}(0) - \tan^{-1}(-\infty)\right] \qquad \Big| \qquad \lim_{t\to\infty} \left[-\tan^{-1}(0) + \tan^{-1}(\infty)\right]$$

$$0 + \frac{\pi}{2} = \frac{\pi}{2} \qquad \Big| \qquad 0 - \frac{\pi}{2} \Rightarrow +\frac{\pi}{2}$$

$$\rightarrow \frac{\pi}{2} + \frac{\pi}{2} = \pi \quad \text{con}$$

II) Improper Integral with discontiunity points:

$$\int_{0}^{1} \frac{1}{x}\,dx \;;\; \int_{-1}^{1} \frac{1}{(x-1)(x+1)} \;;\; \int_{0}^{1} \ln(x)\,dx$$

$$\int_{2}^{3} \log_{10}(x-2)\,dx$$

أحد حدود التكامل ليس صفر مقام أو
سالب تحت جذر غير معرف Imposper
أو الفترة بين حدي التكامل.

* $\int_{a}^{b} f(x)\,d(x) \longrightarrow \lim_{t\to a^+} \int_{t}^{b} f(x)\,dx$

$f(x)$ is discontinuos at $x=a$.   خارج الفترة a^- a^+ b^- b^+ خارج الفترة   (a, b)

* $\int_{a}^{b} f(x)\,dx \longrightarrow f(x)$ is discontinuos at $x=b$

$$\int_{a}^{b} f(x)\,dx \rightarrow \lim_{t\to b^-} \int_{a}^{t} f(a)\,dx.$$

example:

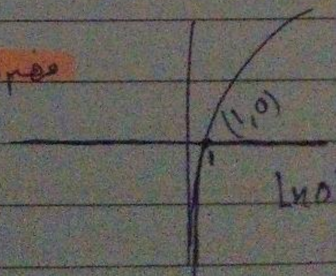
1) $$\int_{0}^{1} \frac{1}{x}\,dx \rightarrow \lim_{t\to 0^+} \int_{t}^{1} \frac{1}{x}\,dx \longrightarrow \lim_{t\to 0^+} \ln x \Big|_{t}^{1} \longrightarrow \lim_{t\to 0^+} (\ln 1 - \ln 0^+)$$

$$= -\ln 0^+$$

$$= \infty$$

diverage

مفهوم رسم سلوك الاقتران



$\ln 0^+ = -\infty$

continue

Subject: ______ / /

Example:

1) $\int_3^4 \frac{1}{x-4}\,dx \rightarrow \lim_{t\to 4^-} \int_3^t \frac{1}{x-4}\,dx \rightarrow \lim_{t\to 4^-} \left(\ln|x-4| \right|_3^t$

$\lim_{t\to 4^-} \left(\ln|t-4| - \ln|3-4| \right)$

$\lim_{t\to 4^-} \ln|t-4| - \ln 1$

$\lim_{t\to 4^-} \ln 4^- - 4 \rightarrow |0^-| = 0^+$

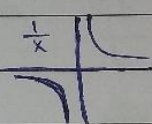
$\ln 0^+ \rightarrow -\infty$ div

2) $\int_4^5 \frac{1}{(x-5)^3}\,dx \rightarrow \lim_{t\to 5} \int_4^t \frac{1}{(x-5)^3}\,dx$

$\lim_{t\to 5} \left. \frac{(x-5)^{-2}}{-2} \right|_4^t \rightarrow \frac{-1}{2} \lim_{t\to 5^-} \left(\frac{1}{(x-5)^2} \right. \rightarrow \frac{-1}{2} \left(\frac{1}{5^- - 5} \right)$

$\frac{-1}{2} \left(\frac{1}{0^+} \right) \rightarrow \infty$

$\frac{1}{x}$



$-\frac{1}{2} \times \infty = -\infty$ div

3) $\int_0^3 \frac{1}{(x-2)}\,dx \rightarrow \int_0^2 \frac{1}{(x-2)}\,dx + \int_2^3 \frac{1}{(x-2)}\,dx \rightarrow$ continue $\Rightarrow \infty$

4) $\int_{-3}^{3} \frac{1}{(x^2-9)}\,dx \rightarrow \lim_{t\to -3^+} \int_t^0 \frac{1}{(x^2-9)} + \lim_{t\to} \int_0^3 \frac{1}{x^2-9}$ *وكمله على الدفتر*

$\int \frac{1}{x^2-9}\,dx = \int \frac{A}{(x-3)} + \int \frac{B}{(x+3)}$

$A = \frac{1}{6}, \quad B = \frac{-1}{6}$

$\lim_{t\to -3^+} \left[\int_t^0 \frac{\frac{1}{6}}{(x-3)}\,dx + \int_t^0 \frac{\frac{-1}{6}}{(x+3)} \right]$

$\lim_{t\to -3^+} \left(\frac{1}{6} \ln|x-3| \Big|_t^0 - \frac{1}{6} \ln|x+3| \Big|_t^0 \right.$

$\lim_{t\to -3^+} \left(\frac{1}{6} \left(\ln|0-3| - \ln|t-3| \right. \right.$

$- \frac{1}{6} \left(\ln|0+3| - \ln|t+3| \right|$

$= \frac{1}{6} \left(\ln 3 - \ln|-3^+ - 3| \right.$

$- \frac{1}{6} \left(\ln|3| - \ln|-3^+ + 3| \right) \rightarrow \frac{1}{6} (\ln 3 - \ln 6) - \frac{1}{6} \left(\ln|3| - \infty \right)$

continue

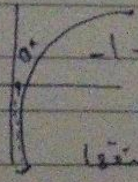
Subject:

Ex: $\int_0^1 \ln x \, dx \rightarrow \lim_{t \to 0^+} \int_t^1 \ln x \, dx \rightarrow \lim_{t \to 0^+} (x \ln x - x) \Big|_t^1$

$\lim_{t \to 0^+} [(1 \cdot \overset{0}{\ln(1)} - 1) - (0^+ \cdot \ln(0)^+ - 0^+)]$

$-1 - (0^+ \cdot \ln 0^+) - 0$

$\ln 0^+ = -\infty$



$-1 - (0^+ \cdot -\infty)$

$-1 - (0) = \boxed{-1}$ con

[illegible]

L'Ho Rule:

1) $\lim_{x \to \infty} \frac{x}{4x+1} \rightarrow \frac{\infty}{\infty}$

$= \frac{1}{4}$

2) $\lim_{x \to 4} \frac{x-4}{x^2-16} = \frac{0}{0}$

$\lim_{x \to 4} \frac{1}{2x} = \frac{1}{8}$

3) $\lim_{t \to 0^+} t \ln t$

$\lim_{t \to 0^+} \quad \lim \frac{\ln t}{\frac{1}{t}} \rightarrow \lim_{t \to 0^+} \frac{\frac{1}{t}}{\frac{-1}{t^2}}$

$\lim_{t \to 0^+} -\frac{\cancel{t^2}}{\cancel{t}} = 0$

Example:

1) $\int_2^3 \ln(x-2) \, dx \longrightarrow \lim_{t \to 2^+} \int_t^3 \ln(x-2) \, dx$

$\because$ Improper علي حسب التكامل محدود بال

$* \int_3^4 \ln(x-2) dx$ not improper.

$\lim_{t \to 2^+} [x \ln(x-2) - x - 2 \ln|x-2|] \Big|_t^3$

$\lim_{t \to 2^+} [3 \ln|\overset{zero}{\cancel{3-2}}| - 3 - 2 \ln|\overset{zero}{\cancel{3-2}}| - \boxed{\infty \cdot = 0^+}$

$(t \ln(t-2) - t - 2 \ln|t-2|)]$

$-3 - \lim_{t \to 2^+} t \ln(t-2) + \lim_{t \to 2^+} t + 2 \lim_{t \to 2^+} \ln|t-2|$

$-3 - (2^+ \ln|2^+ - 2|) + 2 + 2 \cdot (-\infty))$

$\cancel{-3} + \infty \quad + \cancel{2} + 2(-\infty)$ con

-1

$* \int \underbrace{\ln(x-2)}_{u} \underbrace{dx}_{dv}$

$u = \ln(x-2) \quad du = \frac{1}{(x-2)}$

$dv = dx \quad v = x$

$\int \ln(x-2) = x \ln(x-2) - \int \frac{x}{x-2}$

$\int \ln(x-2) = x \ln(x-2) - \left(\int 1 + \frac{2}{(x-2)} \right)$

$x - 2 \overline{) \; x \;}$ 1

$\frac{-x+2}{2}$

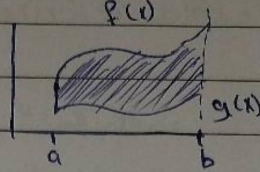
التكامل $x \ln(x-2) - x + 2 \ln|x-2|$

x - in general

Subject: chapter 6 Sunday / /

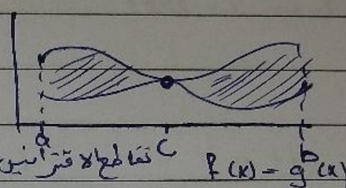
* Applications on integrals:
* Area.
* Volume.

Area $\Rightarrow \int_a^b (f(x) - g(x))\,dx$



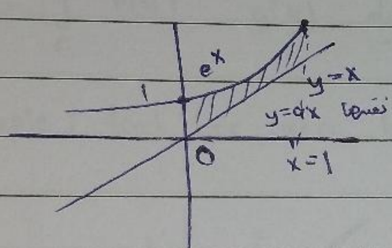
Area $\Rightarrow \int_a^b (f(x) - g(x))\,dx$

$a + \int_c^b (g(x) - f(x))\,dx$



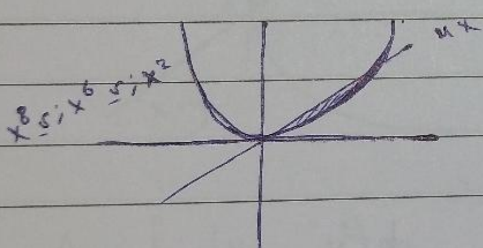
Example: Find the area between:

$y = e^x$ and $y = x$ bounded by $x = 0$ and $x = 1$



$\text{Area} = \int_0^1 (e^x - x)\,dx = \left(e^x - \frac{x^2}{2}\right)\Big|_0^1$

Example: Find the area between $y = x^2$ and $y = 4x$



$A = \int_0^4 (4x - x^2)\,dx$

$\left(2x^2 - \frac{x^3}{3}\right)\Big|_0^4$

اللي فوق - اللي تحت

ساويهم منها اطلع الحدود

$x^2 = 4x$

$x^2 - 4x = 0$

$x = 0 \quad , \quad x =$

Example: find the area between $y = \cos x$, $y = \sin x$ between $x = 0$,